Low Dimensional Topology and Geometric Group Theory
Topologie en basse dimension et théorie géométrique des groupes
(Org: **Adam Clay** (Manitoba), **Mark Powell** (UQAM) and/et **Piotr Przytycki** (McGill))

HANS BODEN, McMaster
The SU(N) Casson-Lin invariants for links

In 1984, Casson constructed an invariant for homology 3-spheres by performing a signed count of irreducible SU(2) representations, and he applied the invariant to the Hauptvermutung in dimension four. In 1992, X.-S. Lin defined a closely related invariant for knots by counting irreducible traceless SU(2) representations of the knot group. Both invariants admit gauge theoretic interpretations in terms of instanton Floer homology and Instanton Knot homology.

In this talk, I will give a brief survey of the SU(2) Casson-Lin invariant for knots and links, as defined by Lin and Harper–Saveliev, respectively. I will then discuss joint work with E. Harper on the SU(N) Casson-Lin invariant of links. The invariants are defined as a signed count of irreducible projective SU(N) representations of the link group, and key to their definition are certain compactness and irreducibility results. Time permitting, I will present computations of the SU(3) Casson-Lin invariant for the Borromean rings, which represents recent joint work with C. Herald.

CHRISTOPHER DAVIS, University of Wiscoinsin at Eau Claire
A genus one algebraically slice knot is 1-solvable.

Joint with Jung Hwan Park, Carolyn Otto and Taylor Martin

Cochran, Orr and Teichner developed a filtration of the knot concordance group indexed by half integers called the solvable filtration. In particular $\mathcal{F}_{0.5}$ is the set of all algebraically slice knots. It has been shown that $\mathcal{F}_n/\mathcal{F}_{n.5}$ is a very large group for $n \geq 0$. The proof of this fact relies on the study of a link associated to an algebraically slice knot called a derivative.

Nothing is known about the other half of the filtration, $\mathcal{F}_{n.5}/\mathcal{F}_{n+1}$. In this project we present a technique which replaces a genus 1 algebraically slice knot by a concordant knot but replaces a derivative with a 0-solvable knot. We will also discuss the application of this technique to higher genus knots, proving that many algebraically slice knots are 1-solvable.

MOON DUCHIN, Tufts

DAVID DUNCAN, McMaster University
Gauge theoretic invariants of surface products

In Donaldson theory, one counts instantons to obtain 4-manifold invariants. There is a similar 4-manifold invariant where one counts holomorphic curves in a certain representation variety. I will outline a proof that these invariants are identical, and discuss some applications to low-dimensional topology.

TULLIA DYMARZ, University of Wisconsin, Madison
Nonrectifiable Delone sets in amenable groups

In 1998 Burago-Kleiner and McMullen constructed the first examples of coarsely dense and uniformly discrete subsets of $\mathbb{R}^n$ that are not biLipschitz equivalent to the standard lattice $\mathbb{Z}^n$ for $n \geq 2$. We will show how to find such sets inside certain other solvable Lie groups. The techniques involve combining ideas from Burago-Kleiner with quasi-isometric rigidity results from geometric group theory.

ELISABETH FINK, University of Ottawa
Morse geodesics in lacunary hyperbolic groups

A geodesic is Morse if quasi-geodesics connecting points on it stay uniformly close. Such geodesics mark hyperbolic directions in the Cayley graph of a group. I will use combinatorial tools to study the geometry of lacunary hyperbolic graded small cancellation groups and show that they contain Morse geodesics. Further I will outline in a simple example an explicit but longer way to find Morse geodesics in such groups. This is joint work with R. Tessera.

MARK HAGEN, University of Cambridge
Quantifying residual properties of virtually special groups

The subgroup H of the group G is *separable* if for each $g \in G - H$, there exists a finite-index subgroup $G' \leq G$ such that $H \subset G'$ but $g \notin G'$. (When $\{1\}$ is a separable subgroup of G, we say that G is *residually finite*.) A natural question is: what must the index of G' be (in terms of the word-length of g with respect to some finite generating set of G, and reasonable data about H) in order to witness separability of g from H? In the case where $H = \{1\}$, this question is made precise by the *residual finiteness growth* function defined by Bou-Rabee, and more generally there are *separability growth* functions measuring how easy it is to separate elements of G from subgroups in a given class.

Using the *special cube complex* machinery of Haglund-Wise, along with some cubical geometry, we proved, with K. Bou-Rabee and P. Patel, that the residual finiteness growth of a virtually special group (a group with a finite-index subgroup embedding in a right-angled Artin group) is bounded by a linear function of the word length. Patel and I generalized this, quantifying the separability growth function for quasiconvex subgroups of virtually special groups. I will discuss some of the ingredients of the proof and mention some applications. Our results give upper bounds on residual finiteness/separability growth; I will briefly discuss why lower bounds are considerably more difficult to obtain, even in the case where G is a free group.

IAN HAMBLETON, McMaster University
Topological 4-manifolds with right-angled Artin fundamental groups

In this talk I will discuss the classification of closed, topological spin 4-manifolds with fundamental group π of cohomological dimension ≤ 3 (up to s-cobordism). In general we must also assume that π also satisfies certain K-theory and assembly map conditions. Examples for which these conditions hold include the torsion-free fundamental groups of 3-manifolds and all right-angled Artin groups whose defining graphs have no 4-cliques.

JOSEPH HELFER, Stanford University
Counting cycles in labeled graphs

Imagine a graph in which each edge is given an orientation and labeled with a letter a or b. Then, given a word w in those letters, you could try to start somewhere and follow the word w around the graph. If you manage to do this, and end up where you started, then you have made a w-cycle. A variant of the famous Hanna Neumann Conjecture from combinatorial group theory says that in any graph, the number of these w-cycles (for a fixed w) should be bounded by the first Betti number of the graph. I will present a proof of this statement. This is joint work with Dani Wise.

YING HU, UQAM
Left-orderability and cyclic branched covers

A group is called left-orderable if one can put a total order ¡ on the set of group elements so that inequalities are preserved by group multiplication on the left. The left-orderability of 3-manifold groups is closely related to the concepts of L-spaces and taut foliations, as conjectured by Boyer-Gordon-Watson. In this talk, we will discuss the left-orderability of fundamental groups of cyclic branched covers of the three sphere.

JINGYIN HUANG, McGill University
Cubulating groups quasi-isometric to right-angled Artin groups

This is a joint work with B. Kleiner. We are motivated by understanding the quasi-isometry rigidity of right-angled Artin groups, which falls into the broader scheme of Gromov's program for quasi-isometry classification of groups and spaces. Suppose G is a right-angled Artin group with finite outer-automophism group. We show that if H is a finitely generated group quasi-isometric to G, then H acts geometrically on a $CAT(0)$ cube complex X, whose combinatorial structure is closely related to the right-angled building and the Salvetti complex associated with G. If times allows, I will talk about how does our cubulation lead to some quasi-isometry rigidity results.

KATARZYNA JANKIEWICZ, McGill University
Cubulations of Artin groups

This is joint work with Jingyin Huang and Piotr Przytycki. A group is cocompactly cubulated if it acts properly and cocompactly by combinatorial automorphisms on a CAT(0) cube complex. We give a characterization of cocompactly cubulated 2-dimensional or 3-generator Artin groups in terms of their defining graphs. Moreover, any such Artin group has a cocompactly cubulated finite index subgroup exactly when it is cocompactly cubulated without passing to a finite index subgroup. In my talk, I will discuss this result and give an overview of the proof.

ADAM LEVINE, Princeton University
Non-orientable surfaces in 4-manifolds

We study the minimal genus problem for embeddings of closed, non-orientable surfaces in a homology cobordism between rational homology spheres or in a closed, definite 4-manifold, using obstructions derived from Heegaard Floer homology. For instance, we show that if a non-orientable surface embeds essentially in the product of a lens space with an interval, its genus and normal Euler number are the same as those of a stabilization of a non-orientable surface embedded in the lens space itself. This is joint work with Danny Ruberman and Saso Strle.

EDUARDO MARTINEZ-PEDROZA, Memorial University
Homological isoperimetric inequalities of 2 dimensional complexes.

The homological filling function of a simply connected space is a generalized isoperimetric function describing the minimal volume required to fill a 1–cycle with an 2–chain. In the framework of homological characterizations of relative hyperbolicity, Groves and Manning posed the question of whether a simply connected 2-complex X with a linear homological isoperimetric inequality, a bound on the length of attaching maps of 2-cells and finitely many 2-cells adjacent to any edge must have a fine 1-skeleton. A graph is fine if for every edge e and each integer $n > 0$, the number of circuits of length n containing e is finite. We provide a positive answer to this question. In this talk, I will discuss our main result, give a brief overview of the proof, and state some related open questions.

MATTHIAS NAGEL, McGill
Unlinking information from 4-manifolds

In the talk I will explain how to obtain lower bounds on unlinking numbers using Donaldson's diagonalisation theorem and a generalisation of a theorem of Cochran-Lickorish. The method will be illustrated using a link from Kohn's table whose unlinking number remained undetermined.

Based on joint work with Brendan Owens.

ARUNIMA RAY, Brandeis University
A new family of links topologically, but not smoothly, concordant to the Hopf link

We give new examples of 2–component links with linking number one and unknotted components that are topologically concordant to the positive Hopf link, but not smoothly so – in fact they are not smoothly concordant to the positive Hopf link with a knot tied in the first component. Such examples were previously constructed by Cha–Kim–Ruberman–Strle; we show that our examples are distinct from theirs. This is joint work with Christopher W. Davis

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DALE ROLFSEN, University of British Columbia
Braids, free group automorphisms and orderings.

Emil Artin, who defined the braid groups B_n, showed that there is a faithful representation of B_n in the automorphism group $Aut(F_n)$ of a free group. It's well-known that that F_n can be ordered in such a way as to be invariant under multiplication on the right or left (known as a bi-ordering). In fact there are uncountably many such bi-orderings if $n > 1$. I'll discuss the question of which braids produce automorphisms of F_n which preserve such a bi-ordering. We make the key observation that a braid produces an auto which preserves a bi-ordering if and only if the "augmented closure" (consisting of the usual closure of the braid together with the braid axis) has a bi-orderable fundamental group of its complement.

One reason for interest in this is the theorem that a knot whose group is bi-orderable cannot produce a Heegaard-Floer L-space via surgery. However, the well-known Whitehead link has complement which fibres over the circle and we argue that its complement has bi-orderable group. On the other hand there exist surgeries on the Whitehead link which do produce L-spaces.

This is joint work with Eiko Kin of Osaka University.

DANIEL WOODHOUSE, McGill
Bounded Packing in Cubulated Groups

Let G be a finitely generated group, and let d_G be the word metric with respect to some finite generating set. let H be a subgroup of G. We say that H has *bounded packing* in G if for all $R > 0$, there is an upper bound $M(D)$ on the number of left cosets that are D-close. That is to say that if $g_1H, \ldots, g_{M(D)}H$ are distinct left cosets, then there exists $1 \leq i < j \leq M(D)$ such that $d_G(g_iH, g_jH) > D$. We prove the bounded packing property for any abelian subgroup of a group acting properly and cocompactly on a CAT(0) cube complex. A main ingredient of the proof is a cubical flat torus theorem.