JOHN BAEZ, U. C. Riverside
Categories in Control

Control theory is the branch of engineering that studies dynamical systems with inputs and outputs, and seeks to stabilize these using feedback. Control theory uses 'signal-flow diagrams' to describe processes where real-valued functions of time are added, multiplied by scalars, differentiated and integrated, duplicated and deleted. In fact, these are string diagrams for the symmetric monoidal category of finite-dimensional vector spaces and the monoidal structure is direct sum. Jason Erbele and I found a presentation for this symmetric monoidal category, which amounts to saying that it is the PROP for bicommutative bimonoids with some extra structure.

A broader class of signal-flow diagrams also includes extra morphisms to model feedback. This amounts to working with the symmetric monoidal category where objects are finite-dimensional vector spaces and the morphisms are linear relations. Erbele also found a presentation for this larger symmetric monoidal category. It is the PROP for a remarkable thing: roughly speaking, an object with two commutative Frobenius algebra structures, such that the multiplication and unit of either one and the comultiplication and counit of the other fit together to form a bimonoid.

In electrical engineering we also need a category where a morphism is a circuit made of resistors, inductors and capacitors. Brendan Fong and I proved there is a functor mapping any such circuit to the relation it imposes between currents and potentials at the inputs and outputs. This functor goes from the category of circuits to the category of finite-dimensional vector spaces and linear relations.