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Topological partition relations for countable ordinals

For X, Y topological spaces and natural numbers l, m we write $X \longrightarrow (Y)^2_{l,m}$ if for every l-coloring of $[X]^2$, the unordered pairs of elements of X, there exists $Z \subseteq X$ homeomorphic to Y such that $c \restriction [Z]^2$ takes at most m colors. Recently C. Pina proved that ω^{ω^ω} is the least ordinal γ that satisfies $\gamma \longrightarrow (\omega^2+1)^2_{l,4}$ for all $l \in \mathbb{N}$ and where all ordinals are endowed with the order topology. The key of Pina's result is the use of certain families of finite sets to represent countable ordinals.

Using families of finite sets to represent countable ordinals, we begin our study with ordinals of the form $\omega \cdot k + 1$ with $k \geq 2$. We find that for every countable ordinal γ there exists a 3-coloring of $[\gamma]^2$ that can't be reduced in a copy of $\omega \cdot 2 + 1$. We set out to find for each $m \geq 3$ the least ordinal γ such that for every l we have that $\gamma \longrightarrow (\omega \cdot 2 + 1)^2_{l,m}$. It turns out that if $\gamma \longrightarrow (\omega \cdot 2 + 1)^2_{l,3}$ for every l, then already $\gamma \geq \omega^{\omega^\omega}$. We carry out a similar analysis for ordinals of the form $\omega \cdot k + 1$ with $k > 2$. This is joint work with William Weiss from the University of Toronto.