WENTANG KUO, University of Waterloo
Multi-dimensional Waring's problem in function fields

The classical Waring's problem is to consider the representations of positive integers as sum of kth powers. S. T. Parsell considers the multi-dimensional generalization of Waring's problem. For a positive integer $d \geq 2$, let $\mathcal{M} = \{(i_1, \ldots, i_d) \in \mathbb{N}^d \,\big|\, i_1 + \cdots + i_d = k\}$. For positive integers P and $n_{\mathbf{i}}$ ($\mathbf{i} \in \mathcal{M}$), denote by $R_{s,k,d}(\mathbf{n}, P)$ the number of solutions of the system of equations

$$x_{11}^{i_1} \cdots x_{1d}^{i_d} + \cdots + x_{s1}^{i_1} \cdots x_{sd}^{i_d} = n_{\mathbf{i}} \quad (\mathbf{i} \in \mathcal{M})$$

with $x_{ij} \in \{1, 2, \ldots, P\}$. S. T. Parsell first proves that when $d = 2$ and $s \geq (14/3)k^2 \log k + (10/3)k^2 \log\log k + O(k^2)$, under certain solubility hypothesis, one has $R_{s,k,d}(\mathbf{n}, P) \gg P^{2s-k(k+1)}$.

In this talk, we will discuss the function field analogue of the multi-dimensional generalization of Waring's problem and apply the recent improvement of Vinogradov-type estimates to get the best known result in the function field setting. It is a joint work with Yu-Ru Liu and Xiaomei Zhao.