Differential Geometry
Géométrie différentielle
(Org: **Ailana Fraser** (UBC) and/et **Regina Rotman** (Toronto))

FLORENT BALACHEFF, University of Lille
Length product of homologically independent loops

I will present a generalization of Minkowski's second theorem to Riemannian torus. This is joint work with Steve Karam.

ALMUT BURCHARD, University of Toronto
Ergodic properties of folding maps on spheres

We consider the trajectories of points on the (d-1)-dimensional sphere under certain folding maps associated with reflections. The main result gives a condition for a collection of such maps to produce dense trajectories. At least d+1 directions are required to satisfy the conditions. (Joint work with A. Dranovski and G. R. Chambers.)

GREGORY CHAMBERS, University of Chicago
Existence of homotopies with prescribed Lipschitz constants

Given Riemannian manifolds M and N, consider maps $f : M \to N$ and $g : M \to N$ which are homotopic and L-Lipschitz. Gromov asked the following question: Does there exist a homotopy from f to g which is itself L-Lipschitz? In this talk, I will describe recent work with D. Dotterrer, F. Manin, and S. Weinberger which partially answers this question. I will also outline some interesting applications of our results.

BENOIT CHARBONNEAU, University of Waterloo
Deformation theory of nearly Kähler instantons

In joint work with Derek Harland, we have developed the deformations theory for instantons on nearly Kähler six-manifolds using spinors and Dirac operators. Using this framework we identify the space of deformations of an irreducible instanton with semisimple structure group with the kernel of an elliptic operator. As an application, we show that the canonical connection on three of the four homogeneous nearly Kähler six-manifolds G/H is a rigid instanton with structure group H. In contrast, these connections admit large spaces of deformations when regarded as instantons on the tangent bundle with structure group SU(3).

ALBERT CHAU, University of British Columbia
An existence time estimate for Kahler Ricci flow and applications

In the talk I will discuss an existence time estimate for Kahler Ricci flow on non-compact manifolds, and related a priori estimates. I will discuss applications to the flow of unbounded curvature metrics in general, and also non-negatively curved Kahler metrics on C^n. Connections will be drawn to Yau's uniformization conjecture which states that a complete non-compact Kahler manifold with positive bisectional curvature is biholomorphic to C^n. The talk will is based on joint work with Luen Fai Tam and Ka Fai Li.

KAEL DIXON, McGill
Local-to-global classification of ambitoric 4-manifolds via uniformization.

A manifold is said to be ambitoric if it admits the structure of a toric Kaehler manifold in two ways, with the two structures sharing the torus action and the conformal class of the metric, but not the orientation. These have been studied by Apostolov, Calderbank, and Gauduchon, who provide a local classification. I will discuss how to extend this local classification to a global classification using Kulkarni's principle of uniformization.

ROBERT HASLHOFER, University of Toronto

Weak solutions for the Ricci flow

We introduce a new class of estimates for the Ricci flow, and use them both to characterize solutions of the Ricci flow and to provide a notion of weak solutions of the Ricci flow in the nonsmooth setting. Given a family of Riemannian manifolds, we consider the path space of its space time. Our first characterization says that the family evolves by Ricci flow if and only if a certain sharp infinite dimensional gradient estimate holds for all functions on path space. We prove additional characterizations in terms of the regularity of martingales on path space, as well as characterizations in terms of log-Sobolev and spectral gap inequalities for a family of Ornstein-Uhlenbeck type operators. Our estimates are infinite dimensional generalizations of much more elementary estimates for the linear heat equation, which themselves generalize the Bakry-Emery-Ledoux estimates for spaces with lower Ricci curvature bounds. Based on our characterizations we can define a notion of weak solutions for the Ricci flow. This is joint work with Aaron Naber.

NIKY KAMRAN, McGill University

Lorentzian Einstein metrics with prescribed conformal infinity

I will present joint work with Alberto Enciso (ICMAT, Madrid) in which we show that given a sufficiently small perturbation g of the conformal metric on the timelike boundary of the $(n+1)$-dimensional anti-de Sitter space at timelike infinity, there exists a Lorentzian Einstein metric on $(-T,T)\times B_n$ whose conformal boundary geometry is given by g.

VITALI KAPOVITCH, University of Toronto

On dimensions of tangent cones in limit spaces with lower Ricci curvature bounds

We show that if X is a limit of n-dimensional Riemannian manifolds with Ricci curvature bounded below and γ is a limit geodesic in X then along the interior of γ same scale measure metric tangent cones $T_{\gamma(t)}X$ are Hölder continuous with respect to measured Gromov-Hausdorff topology and have the same dimension in the sense of Colding-Naber.

This is joint work with Nan Li.

SPIRO KARIGIANNIS, University of Waterloo

Octonionic-algebraic structure and curvature of the moduli space of G_2 manifolds

Let M be a compact irreducible G_2 manifold. The moduli space $\mathcal{M}$ of torsion-free G_2 structures on M is a smooth manifold with an affine Hessian structure. Moreover, it carries a symmetric cubic form and a symmetric quartic form. These tensors are closely related to the curvature of the moduli space, and are built using a particular algebraic structure on 2-tensors on M that is closely related to the octonions. I will explain all of these ideas, and hopefully end with a theorem about estimates on the curvature. This is work in progress with Christopher Lin and John Loftin.

ROBERT MCCANN, University of Toronto

Multi- to one-dimensional transportation

We consider the Monge-Kantorovich problem of transporting a probability density on $\mathbf{R}^m$ to another on the line, so as to optimize a given cost function. We introduce a nestedness criterion relating the cost to the densities, under which it becomes possible to uniquely solve this problem, by constructing an optimal map one level set at a time. This map is continuous if the target density has connected support. We use level-set dynamics to develop and quantify a local regularity theory for this map and the Kantorovich potentials solving the dual linear program. We identify obstructions to global regularity through examples.

OVIDIU MUNTEANU, University of Connecticut
Four dimensional Ricci solitons

I will present recent progress on the asymptotic geometry of complete noncompact four dimensional shrinking Ricci solitons. This talk is based on joint work with Jiaping Wang.

ALEXANDER NABUTOVSKY, University of Toronto
Balanced finite presentations of the trivial group and geometry of four-dimensional manifolds

Recently Boris Lishak has constructed a sequence of finite presentations of the trivial group with just two generators and two relations such that the minimal number of applications of relations required to demonstrate that a generator is trivial grows faster than the tower of exponentials of any fixed height of the length of the finite presentation.

I will explain this result and some of its implications to Riemannian geometry of four-dimensional manifolds. For example, for each closed four-dimensional Riemannian manifold M and each sufficiently small positive ϵ the set of isometry classes of Riemannian metrics on M of volume one with the injectivity radius greater than ϵ is disconnected. A similar disconnectedness result holds for sets of Riemannian structures with $\sup|K|diam^2 \leq x$ on each closed four-dimensional manifold with non-zero Euler characteristic providing that x is sufficiently large. (A joint work with Boris Lishak.)

KASRA RAFI, University of Toronto
Infinitesimal Geometry of Thurston's Lipschitz metric on the Teichmüller space.

Teichmüller space can be equipped with a metric using the hyperbolic structure of a Riemann surface, as opposed to the conformal structure that is used to define the Teichmüller metric. This metric, which is asymmetric, was introduced by Thurston and has not been studied extensively. However, it equips Teichmüller space with a distinctive and rich structure. We examine the infinitesimal geometry of this metric. In the case of the punctured torus, we prove a version of Royden's theorem for Teichmüller metric, namely, we show that the metric is totally non-homogenous.

STEPHANE SABOURAU, Université Paris-Est
Sweepouts in Riemannian geometry

I will present applications of sweepouts techniques in geometry.

CATHERINE SEARLE, Wichita State University
Non-negative curvature and torus actions

I will talk about joint work in progress with Christine Escher about isometric torus actions on non-negatively curved, simply-connected Riemannian manifolds.

WILDERICH TUSCHMANN, Karlsruhe Institute of Technology (KIT)
Moduli spaces of nonnegatively curved Riemannian metrics

A fundamental problem in Riemannian geometry is to understand which manifolds admit metrics displaying certain types of curvature characteristics. Of particular importance amongst these characteristics are the various sign-based conditions, for example negative sectional curvature, positive Ricci curvature and so on. Existence issues for positive scalar curvature metrics are reasonably well understood, but the situation for positive Ricci and positive or nonnegative sectional curvature metrics is somewhat less clear. The theory of manifolds with negative sectional curvature is well-developed, however the existence question is far from resolved.

For the most part this existence question has been a primary focus of research. However, there is an equally intruiging secondary question. If a manifold admits a given type of metric, how are such metrics distributed among all possible Riemannian metrics on this object? For example are they rare or common? How 'many' metrics and geometries does a given manifold allow for?

To answer these questions, one usually looks at the space of metrics satisfying various given curvature conditions on the manifold, or its quotient by the group of diffeomorphisms, the so-called moduli space of metrics, and studies its respective properties.

In my talk, I will survey and describe recent progress on these questions, focusing primarily on connectedness properties of moduli spaces of nonnegative sectional curvature metrics.

GUOFANG WEI, UC Santa Barbara
Local Sobolev Constant Estimate for Integral Ricci Curvature Bounds

We obtain a local Sobolev constant estimate for integral Ricci curvature, which enable us to extend several important tools like maximal principle, gradient estimate, heat kernel estimate and L^2 Hessian estimate to manifolds with integral Ricci lower bounds, including the collapsed case. This is joint work with Xianzhe Dai and Zhenlei Zhang.

HAOMIN WEN, University of Notre Dame
Lens rigidity and scattering rigidity in two dimensions

Scattering rigidity of a Riemannian manifold allows one to tell the metric of a manifold with boundary by looking at the directions of geodesics at the boundary. Lens rigidity allows one to tell the metric of a manifold with boundary from the same information plus the length of geodesics. There are a variety of results about lens rigidity but very little is known for scattering rigidity. We will discuss the subtle difference between these two types of rigidities and prove that they are equivalent for a large class of two-dimensional Riemannian. In particular, two-dimensional simple Riemannian manifolds (such as the flat disk) are scattering rigid since they are lens/boundary rigid (Pestov–Uhlmann, 2005).

BURKHARD WILKING, University of Münster
Manifolds with almost nonnegative curvature operator

We show that n-manifolds with a lower volume bound v and upper diameter D bound whose curvature operator is bounded below by $-\varepsilon(n, v, D)$ also admit metrics with nonnegative curvature operator. The proof relies on heat kernel estimates for the Ricci flow and shows that various smoothing properties of the Ricci flow remain valid if an upper curvature bound is replaced by a lower volume bound.