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Weak solutions for the Ricci flow

We introduce a new class of estimates for the Ricci flow, and use them both to characterize solutions of the Ricci flow and to provide a notion of weak solutions of the Ricci flow in the nonsmooth setting. Given a family of Riemannian manifolds, we consider the path space of its space time. Our first characterization says that the family evolves by Ricci flow if and only if a certain sharp infinite dimensional gradient estimate holds for all functions on path space. We prove additional characterizations in terms of the regularity of martingales on path space, as well as characterizations in terms of log-Sobolev and spectral gap inequalities for a family of Ornstein-Uhlenbeck type operators. Our estimates are infinite dimensional generalizations of much more elementary estimates for the linear heat equation, which themselves generalize the Bakry-Emery-Ledoux estimates for spaces with lower Ricci curvature bounds. Based on our characterizations we can define a notion of weak solutions for the Ricci flow. This is joint work with Aaron Naber.