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From the Descent Algebra to the Peak Algebra on the Symmetric Group

Given a permutation σ in the symmetric group of order n , the descents are the positions $\{i \mid \sigma(i) > \sigma(i+1)\}$. This leads to an algebra whose elements are sums of permutations sharing the same descent set. This algebra has been widely studied for its connections with Coxeter groups and card shuffling. In a similar way, the peaks correspond to the set $\{i \mid \sigma(i-1) < \sigma(i) > \sigma(i+1)\}$. This allows us to define a subalgebra of the descent algebra, that is as well a left ideal of it. We extend results on the descent algebra to their equivalent in terms of peaks.