YANNICK BONTHONNEAU, CIRGET - CRM

Continuous and discrete Weyl laws for negatively curved cusp manifolds

The purpose of my talk will be to give a new refinement in the spectral theory of the Laplace operator on manifolds with exact hyperbolic cusps, when the curvature is negative. I will give estimates on some spectral counting functions.

The usual counting function for the point spectrum $N(T)$ is not the relevant quantity as the Laplace operator has continuous spectrum. Instead, we have to consider $N(T)+S(T)$, where $S(T)$ is the phase shift in the appropriate scattering problem. First, when the curvature is negative, we will see that

$$N(T)+S(T)=\frac{\mathrm{vol}(B^*M)}{(2\pi)^n}T^n+aT\log T+bT^{n-1}+\mathcal{O}\left(\frac{T^{n-1}}{\log T}\right)$$

with some constants a and b.

The function $z\mapsto \exp iS(i((n-1)/2-z))$ has a meromorphic extension to $\mathbb{C}$, with its poles contained in $\Re z<(n-1)/2$. These poles can be seen as a discrete spectral set replacing the eigenvalues in the compact case, they are called resonances. I proved that for a generic set of metrics, most of these resonances are contained in a strip $\{\delta<\Re z<(n-1)/2\}$ at high frequency, where $\delta<(n-1)/2$. Under this genericity assumption, we will see that

$$\#\{s \text{ resonance} \mid \Re s>\delta,\ |\Im s|\leq T\}=AT^n+BT^{n-1}+CT\log T+\mathcal{O}\left(\frac{T^{n-1}}{\log T}\right).$$