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Nonlocal attractive-repulsive interaction energies of binary densities

In this talk I will consider the minimization of nonlocal interaction energies of the form

$$E[\rho] = \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} K(x-y)\rho(x)\rho(y) dx dy$$

over the set of admissible functions $\{\rho \in L^1(\mathbb{R}^n, \{0, 1\}) : \|\rho\|_{L^1} = 1\}$ where the interaction potential K is of the form of a power-law with attractive and repulsive components. This type of energies arise naturally in descriptions of systems of interacting particles, as well as continuum descriptions of systems with long-range interactions and are used in modelling collective behavior of many-agent systems, granular media and self-assembly of nanoparticles. The additional nonconvex constraint $\rho \in \{0, 1\}$, on one hand, poses new challenges such as the existence of minimizers. On the other hand, with this constraint the first and second variations of the energy E can simply be expressed explicitly as conditions on the boundary of the set $\{\rho = 1\}$. After establishing the existence of minimizers, I will characterize the ground state when the attraction is given by a quadratic interaction and the repulsion by Newtonian potential, and comment on qualitative properties of local minimizers. This is a joint work with R. Choksi.