We consider the following problem: given a set of algebraic conditions on an $n$-tuple of functions and their first $l$ derivatives, admitting finitely many solutions in a differentially closed field, give an upper bound for the number of solutions. I will present estimates in terms of the degrees of the algebraic conditions, or more generally the volumes of their Newton polytopes (analogous to the Bezout and BKK theorems). The estimates are singly-exponential with respect to $n, l$ and have the natural asymptotic with respect to the degrees or Newton polytopes. This result sharpens previous doubly-exponential estimates due to Hrushovski and Pillay.

I will give a brief overview of the geometric ideas behind the proof. If time permits I will also discuss some diophantine applications.