Integrable Systems: Recent Developments
Les récents développement dans les Systèmes intégrables
(Org: **Stephen Anco** (Brock) and/et **Dmitry Pelinovsky** (McMaster))

XIANGKE CHANG, University of Saskatchewan
A generalized nonisospectral Camassa-Holm equation and its multipeakon solutions

The Camassa-Holm (CH) equation is a well-known example of integrable systems and attracted many researches. The most attractive character is that it admits certain weak solution called peaked solutions (peakons). The explicit expressions of its multi-peakon solutions have been obtained by Beals, Sattinger and Szmigielski, who used inverse spectral method and the Stieltjes theorem on continued fractions. Observing that the expressions can be described by means of Hankel determinants, we reconfirmed this by an algebraic approach. By altering the evolution with respect to time t for the moments of Hankel determinant, after a series of inverse calculations, we propose an extension of the Camassa-Holm equation, which also admits the multipeakon solutions. The novel aspect is that our approach is mainly based on classic determinant technique. Furthermore, the proposed equation is shown to possess a nonisospectral Lax pair.

[1] R. Beals, D. H. Sattinger, and J. Szmigielski. Multi-peakons and a theorem of Stieltjes. Inverse Problems, 15:L1–L4, 1999.

[2] R. Beals, D. H. Sattinger, and J. Szmigielski. Multipeakons and the classical moment problem. Adv. Math., 154(2):229–257, 2000.

[3] X.K. Chang, X.M. Chen, and X.B. Hu, A generalized nonisospectral Camassa-Holm equation and its multipeakon solutions. 263:154–177, 2014

DMITRY KOROTKIN, Concordia University
Bergman tau-function and geometry of moduli spaces

Abstract. Bergman tau-function is a universal object appearing in several seemingly unrelated areas - from Riemann-Hilbert problems of a special type - to asymptotical expansion of matrix integrals and computation of determinant of Laplacian on polyhedral Riemann surfaces. The Bergman tau-function can be defined on various spaces: Hurwitz spaces (spaces of meromorphic functions on Riemann surfaces), spaces of holomorphic 1-differentials and n-differentials on Riemann surfaces etc. The analysis of global properties of the Bergman tau-function on these spaces allows to derive new (and re-derive some important old) relations between natural divisors on these spaces. In particular, we obtain an expression for the Hodge class on spaces of admissible covers, reproduce Mumford's formula relating determinant line bundles of vector bundles of abelian and quadratic differentials over moduli spaces of Riemann surfaces etc.

The talks are based on joint works with A.Kokotov and P.Zograf.

ALEXANDER ODESSKI, Brock University
Non-homogeneous systems of hydrodynamic type possessing Lax representations

We consider 1+1 - dimensional non-homogeneous systems of hydrodynamic type that possess Lax representations with movable singularities. We present a construction, which provides a wide class of examples of such systems with arbitrary number of components. In the two-component case a classification is given. This talk is based on the paper:

A.V. Odesskii, V.V. Sokolov, Non-homogeneous systems of hydrodynamic type possessing Lax representations, Commun. Math. Phys., 324(1), 47-62 (2013),

and some unpublished computations.

DANILO RIGLIONI, Centre de Recherches Mathématiques
Superintegrable systems with spin induced by coalgebra symmetry

A method for deriving superintegrable Hamiltonians with a spin-orbital interaction is presented. The method is applied to obtain a new superintegrable system in Euclidean space with the following properties. It describes a rotationally invariant interaction between a particle of spin $1/2$ and one of spin 0. Its Hamiltonian commutes with total angular momentum ($\vec{J}$) and with additional vector integrals of motion ($\vec{X}$) , ($\vec{Y}$) with components that are third-order differential operators. The integrals of motion form a polynomial algebra under commutation. The system is exactly solvable (in terms of Laguerre polynomials) and the bound state energy levels are degenerate and described by a Balmer type formula. When the spin-orbital potential is switched off the system reduces to a hydrogen atom.

MASOUMEH SAJEDI, Université de Montréal

Two-dimensional superintegrable quantum systems with potentials expressed in terms of Painlevé transcendents

We consider quantum superintegrable Hamiltonians that admit separation of variables in Cartesian coordinates and allow the existence of a fourth-order integral of motion in the two-dimensional Euclidean space. The most interesting ones involve potentials expressed in terms of Painlevé transcendents. We show how the results are related to the third-order superintegrable systems.

VASILISA SHRAMCHENKO, University of Sherbrooke

Poncelet theorem and Painlevé VI

This is a joint work with V. Dragovic (UT Dallas). In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI (1/8,-1/8,1/8,3/8) equation starting with any Poncelet trajectory, that is a closed billiard trajectory inscribed in a conic and circumscribed about another conic. In this talk I will show that Hitchin's construction is nothing but the Okamoto transformation between Picard's solution and the general solution of the Painlevé VI (1/8,-1/8,1/8,3/8) equation. Moreover, this Okamoto transformation can be written in terms of an Abelian differential of the third kind on the associated elliptic curve, which allows to write down solutions to the corresponding Schlesinger system in terms of this differential as well. This solution of the Schlesinger system admits a natural generalization to hiperelliptic curves.

THOMAS WOLF, Brock University

Integrability study of breaking-wave equations

There is considerable interest in the study of equations of the form $u_t - u_{txx} = f(u, u_x, u_{xx}, u_{xxx})$ that describe breaking waves. Two main examples in this class are the Camassa-Holm equation and the Novikov equation, both of which are integrable systems in the sense of having a Lax pair and a bi-Hamiltonian structure as well as hierarchies of local symmetries and local conservation laws. In this talk, I present results from a recent study of the integrability of two interesting new 1-parameter families of breaking-wave equations with power nonlinearities. One family contains the Camassa-Holm equation and the Novikov equation. The other family is a generalization of the Camassa-Holm equation analogous to the generalized KdV equation.