**NOAH SCHWEBER**, UC-Berkeley

*Computability and structures in generic extensions*

Using set-theoretic forcing, many concepts of computable structure theory can be extended to structures of arbitrary cardinality in a natural way. For example, given two possibly uncountable structures $A$ and $B$, say $A$ is *generically Muchnik reducible* to $B$ if, whenever $V[G]$ is a forcing extension in which both $A$ and $B$ are countable, any copy of $B$ with domain $\omega$ in $V[G]$ Turing-computes a copy of $A$. Basic properties of forcing ensure that this translation is well-behaved. In particular, by Shoenfield's Absoluteness Theorem these notions are all independent of the choice of forcing extension, in a precise sense. Having defined generic Muchnik reducibility, it then becomes natural to see how it compares with other notions of relative complexity of uncountable objects - for example, relative constructibility: if $A$ is generically Muchnik reducibile to $B$, are copies of $A$ constructible relative to copies of $B$?

In this talk, we will introduce generic Muchnik reducibility, and examine the question mentioned above. We show that the answer to this question (when made precise) is in general "no," although the answer is "yes" if we assume that $B$ has cardinality at most $\aleph_1$. We will also show how these ideas lead to a new proof of a theorem of Harrington, that a counterexample to Vaught's conjecture must have models of cardinality $\aleph_1$ with Scott rank arbitrarily high below $\omega_2$.

This is joint work with Julia Knight and Antonio Montalban.