Commutative Algebra: Interactions with Algebraic Combinatorics, Algebraic Geometry, and Representation Theory
Algèbre commutative : interactions avec la combinatoire algébrique, la géométrie algébrique et la théorie des représentations
(Org: **Tony Geramita** (Queen's & Genoa) and/et **Adam Van Tuyl** (Lakehead))

HIRO ABO, University of Idaho
Eigenvectors of tensors

The concept of an eigenvector of an $n \times \cdots \times n$ tensor was introduced by L. Qi in 2007. In 2013, G. Ottaviani and L. Oeding showed that the set of eigenvectors of a given $n \times \cdots \times n$ tensor (being considered as points in the projective $(n-1)$-space) can be described as the zero locus of a global section of the (suitably twisted) tangent bundle on the projective $(n-1)$-space. The purpose of this talk is to use this interpretation to describe configurations of eigenvectors of $2 \times 2 \times 2$ tensors in the projective plane. This is part of the on-going project with B. Sturmfels and A. Seigal.

ALI ALILOOEE, Dalhousie University
When is a Squarefree Monomial Ideal of Linear Type?

In 1995 Villarreal gave a combinatorial description of the equations of Rees algebras of quadratic squarefree monomial ideals. His description was based on the concept of closed even walks in a graph. In this talk we will generalize his results to all squarefree monomial ideals.

JENNIFER BIERMANN, Mount Holyoke College
Generalized edge and cover ideals

We generalize the notion of an edge ideal to a t-edge ideal whose minimal monomial generators consist of a vertex and t of its neighbors. We study the algebraic properties of the t-edge ideal of a graph as well as the combinatorial properties of its associated Stanley-Reisner simplicial complex.

MATS BOIJ, KTH - Royal Institute of Technology
Cones of Hilbert Functions

In a joint work with Gregory G. Smith we study the closed convex hull of various collections of Hilbert functions. In this work we focus on graded modules generated in degree zero over a standard graded polynomial ring. In this context, we completely describe the supporting hyperplanes and extreme rays for the cones generated by the Hilbert functions of all modules, all modules with bounded a-invariant, and all modules with bounded Castelnuovo-Mumford regularity. The first of these cones is infinite-dimensional and simplicial, the second is finite-dimensional but neither simplicial nor polyhedral, and the third is finite-dimensional and simplicial. We also give explicit linear bounds for the graded Betti numbers of modules with a given Hilbert function in the case of bounded Castelnuovo-Mumford regularity.

ENRICO CARLINI, Politecnico di Torino and Monash University
On the Waring rank of cubics

In this talk I will present a new approach to the study of the Waring rank of homogeneous polynomials. This method is particularly useful in the case of degree three forms. For example, we can complete the description of the possible ranks for reducible cubics. This is a joint work with Cheng Guo (University of Technology Sydney) and Emanuele Ventura (Aalto University).

LUCA CHIANTINI, Universita' di Siena, Italy
On the representation of general forms

I will illustrate some recent results on the representation of polynomials by means of determinants and pfaffians of matrices of forms, with fixed degrees. One can always represent a general ternary forms as a determinant or a pfaffian, whatever the prescribed degrees are, but this property fails for form of high degree in more variables. Hence, I will consider the minimal number of determinants or pfaffians needed to represent forms with more than three variables. This problem, of clear algebraic flavor, can be studied with methods of Algebraic geometry (secant varieties), and it turns out to have some intersection with applications in control theory.

SUSAN COOPER, North Dakota State University
Fat Points, Grids, and Partial Intersections

It is well-known that characterizing Hilbert functions of fat points is an open and very difficult problem. One approach is to compare the Hilbert functions of these non-reduced schemes to those of well-known families of reduced point sets. In this talk we will look at connections between Hilbert functions of fat points supported inside grid complete intersections and Hilbert functions of reduced point sets called partial intersections. This is joint work in progress with E. Guardo.

NURSEL EREY, Dalhousie University
Multigraded Betti numbers of monomial ideals

In this talk, we give a class of facet covers that if present, ensure the existence of a Betti number for any monomial ideal. In the case of facet ideals of simplicial forests, all Betti numbers are described by such covers.

ELENA GUARDO, Università di Catania
On the Hilbert functions of points in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

Let H_X be the trigraded Hilbert function of a set X of reduced points in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$. We show how to extract some geometric information about X from H_X. This is a joint paper with A. Van Tuyl

TAI HA, Tulane University
Algebraic invariants of fiber products

Let X and Y be varieties over a field k, and let $Z = X \times_k Y$ be their fiber product. In this talk, we consider the question of how singularity and multiplicity theories of Z can be understood from those of X and Y. Specifically, our focus will be on the Castelnuovo-Mumford regularity and depth of symbolic and regular powers of their defining ideals.

BRIAN HARBOURNE, University of Nebraska-Lincoln
Survey of recent work on resurgences and the containment problem

As of February, 2013, it is known that the square of an ideal of points in the projective plane need not contain the symbolic cube of the ideal. New examples have recently been found (arXiv:1407.2966: Bauer, Di Rocco, Harbourne, Huizenga, Lundman, Pokora, Szemberg), but still all known examples are related to the singular points of configurations of lines. I will summarize what is known and what is not known and discuss recent work on computing resurgences of these ideals (see arXiv:1404.4957: Dumnicki, Harbourne, Nagel, Seceleanu, Szemberg, Tutaj-Gasińska; also see arXiv:1410.0312: Seceleanu).

PAOLO MANTERO, University of California at Riverside
On a conjecture by G. V. Chudnovsky

A long-standing conjecture of G. V. Chudnovsky predicts lower bounds for the minimal degree of a hypersurface passing through a set of n fixed (simple distinct) points in the projective space $\mathbb{P}^N$ with multiplicity m. Except for $\mathbb{P}^2$ and $\mathbb{P}^3$, the conjecture is wide-open, and so far the best lower bound for $N > 3$ was proved by Esnault-Viehweg in 1983. Chudnovsky's conjecture is implied by a recent (wide-open) conjecture posed by Harbourne and Huneke about inclusions between symbolic and ordinary powers of ideals.

In this talk, based on joint work with L. Fouli and Y. Xie, we prove Chudnovsky's conjecture for "most" sets of n points in $\mathbb{P}^N$.

SARAH MAYES, Quest University Canada
Boij-Söderberg decompositions of systems of ideals

Since the existence of Boij-Söderberg decompositions of Betti tables was proven in 2007, much work has been done to understand them. However, we still know little about the meaning of these decompositions and the patterns that exist within them. In this talk, we will discuss some results and conjectures related to the asymptotic stability of Betti tables of systems of ideals and of their Boij-Söderberg decompositions.

JUAN MIGLIORE, University of Notre Dame
Secant varieties to the varieties of reducible hypersurfaces

Let $S = k[x_1, \ldots, x_n]$ be the standard graded polynomial ring, where k is an algebraically closed field. Let $\lambda = [d_1, \ldots, d_r]$ be a partition of a positive integer d into $r \geq 2$ parts. In $\mathbb{P}^{N-1}$, where $N = \binom{d+n-1}{n-1}$, we have $\mathbb{X}_{n-1,\lambda}$, the *variety of reducible hypersurfaces (forms) of type* λ. The dimension of $\mathbb{X}_{n-1,\lambda}$ is well known, and there is a well-known formula for the expected dimension of the variety $\sigma_\ell(\mathbb{X}_{n-1,\lambda})$ spanned by the secant $\mathbb{P}^{\ell-1}$'s to $\mathbb{X}_{n-1,\lambda}$ in $\mathbb{P}^{N-1}$. When this expected dimension is not achieved, $\sigma_\ell(\mathbb{X}_{n-1,\lambda})$ is said to be *defective*. We compute the precise dimension of $\sigma_\ell(\mathbb{X}_{n-1,\lambda})$ in many new cases, identifying the instances when $\sigma_\ell(\mathbb{X}_{n-1,\lambda})$ is defective. We furthermore give a conjecture that, if true, would explicitly give the precise dimension of $\sigma_\ell(\mathbb{X}_{n-1,\lambda})$ in all cases. This conjecture is based on the Weak Lefschetz Property for a certain collection of graded artinian algebras. This is joint work in progress with M. Catalisano, A.V. Geramita, A. Gimigliano, B. Harbourne, U. Nagel and Y.S. Shin.

ALESSANDRO ONETO, Stockholm University
A Waring problem with higher degree terms

The aim of this talk is to present a variant of the traditional Waring problem for polynomials. We define the k^{th}-Waring rank of a form of degree kd to be the length of its minimal additive decomposition as sum of k^{th}-powers of forms of degree d. After the presentation of the main result due to Fröberg, Ottaviani and Shapiro who gave an upper bound (asymptotically sharp for large d) for the k^{th}-Waring rank of a generic form, we will see other recent results in the case of sum of squares and monomials. This is a joint work with Enrico Carlini.

YONG SU SHIN, Sungshin Women's University
The Minimal Free Graded Resolution of A Star-Configuration in $\mathbb{P}^n$

We find the minimal free graded resolution of the ideal of a star-configuration in $\mathbb{P}^n$ of type (r, s) defined by general forms in $R = \Bbbk[x_0, x_1, \ldots, x_n]$. This generalises the result of Ahn and Shin from a specific value of $r = 2$ to any value of $1 \leq r \leq \min\{n, s\}$, and that of Geramita, Harbourne, and Migliore from a linear star-configuration in $\mathbb{P}^n$ to a star-configuration in $\mathbb{P}^n$. Moreover, we show that any star-configuration in $\mathbb{P}^n$ is arithmetically Cohen-Macaulay.

GREGORY G. SMITH, Queen's University
Parliaments of polytopes and toric vector bundles

To each torus-equivariant vector bundle over a smooth complete toric variety, we associate a collection of rational convex polytopes, called a parliament of polytopes. We will explore the correspondence between features of the toric vector bundle and properties of the parliament of polytopes. This talk is based on joint work with Sandra Di Rocco and Kelly Jabbusch.

TOMASZ SZEMBERG, Pedagogical University of Cracow
On Sylvester-Gallai Theorem for conics

The Sylvester-Gallai Theorem asserts that given non-collinear points in the real projective plane, there is a line passing through exactly two of the given points. This has been generalized in the following way for curves of degree two by Wiseman and Wilson: given a finite set of points in the real projective plane either all points are contained in a conic, or there exists a conic passing through exactly five of the given points and this conic is unique (i.e. it is determined by these five points). I will show a new proof of this result using methods from algebraic geometry and I will discuss some further generalizations and present a couple of open problems.

ZACH TEITLER, Boise State University
Lower bound for ranks of invariant forms

We give a lower bound for the Waring rank and cactus rank of forms that are invariant under an action of a connected algebraic group. We use this to improve the Ranestad–Schreyer–Shafiei lower bounds for the Waring ranks and cactus ranks of determinants of generic matrices, Pfaffians of generic skew-symmetric matrices, and determinants of generic symmetric matrices. This is joint work with Harm Derksen.

DAVID WEHLAU, Royal Military College
Hilbert Functions of Graded Gorenstein Ideals with an S_n-action.

We are interested in describing all possible Hilbert Functions of graded artinian Gorenstein quotients of the polynomial ring which are also representations of the symmetric group. Bergeron, Garsia and Tesler gave an explicit description of the graded characters of all such algebras whose socle is spanned by an alternating function. This leaves only the case where the socle is spanned by a symmetric function. We will determine the graded characters and Hilbert functions for these quotients for a large class of symmetric functions. We do this by relating these algebras to subrepresentations of the regular representation of the symmetric group. These graded characters turn out to be closely related to Kostka-Foulkes polynomials.

This is joint work with A. Geramita(Queen's University) and A. Hoefel(Google).