Arithmetic Algebraic Geometry
Géométrie algébrique arithmétique
(Org: **Manfred Kolster** (McMaster))

DYLAN ATTWELL-DUVAL, McGill University
Counting cusps for orthogonal Shimura varieties

A rational vector space (V,q) of signature $(2,n)$ gives rise to an orthogonal Shimura variety. Due to accidental isomorphisms of the orthogonal group in low dimensions, many special cases of such varieties are already well known through different reductive groups. In this talk we shall discuss the structure of the Baily-Borel compactifications of these spaces, particularly the case when V splits two hyperbolic planes over $\mathbb{Q}$, which holds for all V if $n \geq 5$. We find that arithmetic subgroups arising from maximal lattices yield explicit formulas. These results are a portion of my Ph.D. work.

ERNST KANI, Queen's University
Perfect Cuboids and the Box Variety

A perfect cuboid is a rectangular box (cuboid) whose edges, face diagonals and body diagonal all have integer length. It is an old open problem (perhaps dating back to Euler or before) whether there are any perfect cuboids. It is also unknown whether there can be at most finitely many perfect cuboids.

The box variety is an explicit algebraic surface in 6-dimensional projective space whose points with positive integer coordinates correspond precisely to the perfect cuboids. In the last few years several people (Beauville, Freitag, Salvati Manni, Stoll, Testa and others) have studied the geometric structure of the box variety, and this sheds new insight into the above open problems.

In my talk I will first discuss some early history of perfect cuboids. Then I will explain what is known about the box variety, and how this relates to the open problems.

ANTONIO LEI, Université Laval
Universal norm of crystalline classes

Let T be a crystalline p-adic representation of G_F, where F is a number field in which p is unramified. We may define a Selmer group of T using Bloch-Kato's crystalline classes. Given a family of crystalline classes over the cyclotomic extension of F that satisfy certain compatibility conditions, I will talk about how to interpolate these classes by power series using Perrin-Riou exponential map.

MENG FAI LIM, University of Toronto
Comparing the Iwasawa mu-invariants of Selmer groups

The main conjecture of Iwasawa theory is a conjecture on the relation between a Selmer group, which is a module over an (not necessarily commutative) Iwasawa algebra, and a conjectural p-adic L-function. This p-adic L-function is in turn expected to satisfy a conjectural functional equation in a certain sense. In view of the main conjecture and this functional equation, one would expect to have certain algebraic relationship between the Selmer group attached to a Galois representation and the Selmer group attached to the Tate twist of the dual of the Galois representation which can be thought as an algebraic manifestation of the functional equation. It is precisely a component of this algebraic relationship that this talk will cover. Namely, we will show that the Selmer group attached to a Galois representation and the Selmer group attached to the Tate twist of the dual representation have the same generalized Iwasawa μ-variant. If time permits, we will also mention how the technique used in this study may also be applied to another context; namely, to compare the Iwasawa μ-variant of Selmer groups of congruent Galois representations. This latter comparison is motivated by the philosphy that the main conjecture should be preserved under congruence.

KUMAR MURTY, Toronto

JONATHAN SANDS, University of Vermont
Zeta-functions and Finiteness of the Number of Ideal Classes in Quaternion Orders

Inspired by Stark's analytic proof of the finiteness of class numbers of rings of integers in algebraic number fields, we provide an alternative, analytic proof of the finiteness of the number of classes of left ideals in a maximal order of a division quaternion algebra over a number field.

ROMYAR SHARIFI, University of Arizona
The arithmetic of modular symbols

That the geometry of modular curves has something to say about the arithmetic of cyclotomic fields has long been known. Out of the 2-dimensional Galois representation attached to a level p newform congruent to an Eisenstein series modulo p, Ribet constructed a p-torsion subgroup of the class group of the pth cyclotomic field $\mathbb{Q}(\mu_p)$. In their proof of the Iwasawa main conjecture, Mazur and Wiles similarly found the entire minus part of the p-part of the class group of any cyclotomic field. We conjecture a more precise relationship: the quotient by an Eisenstein ideal of the space of cusp forms of some level N should be isomorphic to a cohomology group closely related to the class group of $\mathbb{Q}(\mu_N)$ via a very simple map ϖ taking modular symbols to cup products of cyclotomic units. This map ϖ has a conjectural inverse Υ constructed from modular representations. Fukaya and Kato have proven a major result towards this conjecture. We intend to describe both the conjecture and the best understood of many hoped for analogues, which is to say for the function field $\mathbb{F}_q(t)$ in place of $\mathbb{Q}$. The work on this analogue is joint with Takako Fukaya and Kazuya Kato.

AL WEISS, University of Alberta
What is equivariant Iwasawa theory?

The Main Conjecture of classical Iwasawa theory was proved by Wiles (1990). Equivariant Iwasawa theory is a generalization/refinement of this with a 'main conjecture' ('emc') which was designed to apply to Chinburg's root number conjecture and its subsequent generalizations. In this talk, we discuss the formulation of the 'emc' and the ideas involved in the proof, with J. Ritter, of a weak form of it (2010). We then briefly discuss the gap remaining between the 'emc' and this weak form.

SOROOSH YAZDANI, Google Inc. and University of Waterloo
Largest Prime Divisor of terms of EDS

In 1965 Erdos conjectured that

$$\liminf_{n\to\infty}\frac{P(2^n-1)}{n}=\infty,$$

where $P(x)$ is the largest prime divisor of x. Thirty five years later, Murty and Wong proved that if ABC conjecture is true then for $\varepsilon>0$ and for positive integers $a>b$ we have $P(a^n-b^n)>n^{2-\varepsilon}$ for sufficiently large n. Recently, Stewart proved Erdos's conjecture unconditionally using linear forms of logarithms for the terms in Lucas sequence.

Followin Murty and Wong, we investigate the largest prime divisor of the terms in an Elliptic Divisibility Sequence.

This is joint work with Amir Akbary.