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The arithmetic of modular symbols

That the geometry of modular curves has something to say about the arithmetic of cyclotomic fields has long been known. Out of the 2-dimensional Galois representation attached to a level p newform congruent to an Eisenstein series modulo p, Ribet constructed a p-torsion subgroup of the class group of the pth cyclotomic field $\mathbb{Q}(\mu_p)$. In their proof of the Iwasawa main conjecture, Mazur and Wiles similarly found the entire minus part of the p-part of the class group of any cyclotomic field. We conjecture a more precise relationship: the quotient by an Eisenstein ideal of the space of cusp forms of some level N should be isomorphic to a cohomology group closely related to the class group of $\mathbb{Q}(\mu_N)$ via a very simple map ϖ taking modular symbols to cup products of cyclotomic units. This map ϖ has a conjectural inverse Υ constructed from modular representations. Fukaya and Kato have proven a major result towards this conjecture. We intend to describe both the conjecture and the best understood of many hoped for analogues, which is to say for the function field $\mathbb{F}_q(t)$ in place of $\mathbb{Q}$. The work on this analogue is joint with Takako Fukaya and Kazuya Kato.