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The integrable nature of modulational instability

We investigate the nonlinear stage of the modulational instability (MI) by characterizing the IVP for the focusing NLS equation with non-zero boundary conditions (NZBC) at infinity, using the recently formulated inverse scattering transform (IST) for this problem. While the linearization of NLS ceases to be valid when perturbations have grown sufficiently large, the IST holds at all times, and therefore provides the best way to study the nonlinear stage of MI. First we study the scattering problem with piecewise constant ICs which are generalizations to NZBC of a potential well and barrier, and we obtain several results. We prove that there are arbitrarily small perturbations of the constant background for which there are discrete eigenvalues, which shows that no area theorem is possible for the NLS equation with NZBC. We prove that there there is a large class of perturbations for which no discrete eigenvalues are present, which shows that solitons cannot be the primary vehicle for the manifestation of MI, contrary to a recent conjecture. Finally, we compute the small-deviation limit of IST and we compare it with the linearization of NLS. This allows us to identify the precise nonlinear analogue of the unstable Fourier modes within the IST. These are the Jost eigenfunctions for values of the scattering parameter in a finite interval of the imaginary axis around the origin. Importantly, this shows that the IST contains an automatic mechanism for the saturation of the MI.