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Expansions of the ordered additive group of real numbers by two discrete subgroups

Let $a \in \mathbb{R}$. We consider the following structure $\mathcal{R}_a := (\mathbb{R}, <, +, \mathbb{Z}, \mathbb{Z}a)$. Although it is well known that $(\mathbb{R}, <, +, \mathbb{Z})$ has a decidable theory and other desirable model theoretic properties (arguably due to Skolem and later rediscovered independently by Weispfenning and Miller), the question whether the theory of $\mathcal{R}_a$ is decidable even for some irrational number a has been open for a long time. The interest in these structures arises among other things from the observation that the structure $\mathcal{R}_a$ codes many of the Diophantine properties of a. In this talk, I will show that when a is quadratic, the theory of $\mathcal{R}_a$ is decidable. The proof of this statement depends crucially on the periodicity of the continued fraction expansion of a and combines classical tools from the theory of Diophantine approximations (in particular, Ostrowski representations) with Büchi's celebrated theorem about the decidability of the monadic second order theory of one successor.