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Varieties and isogeny classes of elliptic curves

Fix some transcendental numbers $\bar{a} \in \mathbb{C}^n$. Let $Iso(\bar{a})$ denote the isogeny class of $\bar{a}$, viewing $\mathbb{A}^n$ as the moduli space of products of elliptic curves. Call a variety weakly special if it is defined by a Boolean combination of modular polynomial relations and equations of the form $x_i = b$.

If $V \subseteq \mathbb{C}^n$ is a non-weakly-special variety, then $V \cap Iso(\bar{a})$ is not Zariski dense in V. We will discuss how to use differential algebra to give an effective upper bound on the degree of the Zariski closure of $V \cap Iso(\bar{a})$. In the case that one knows that $V \cap Iso(\bar{a})$ is zero-dimensional (e.g. V is a curve or V contains no weakly special varieties), this gives an effective bound on the number of points in the intersection.

The proofs use intersection theory in jet spaces and various notions from geometric model theory.