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Poncelet theorem and Painlevé VI

This is a joint work with V. Dragovic (UT Dallas). In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI $(1/8, -1/8, 1/8, 3/8)$ equation starting with any Poncelet trajectory, that is a closed billiard trajectory inscribed in a conic and circumscribed about another conic. In this talk I will show that Hitchin's construction is nothing but the Okamoto transformation between Picard's solution and the general solution of the Painlevé VI $(1/8, -1/8, 1/8, 3/8)$ equation. Moreover, this Okamoto transformation can be written in terms of an Abelian differential of the third kind on the associated elliptic curve, which allows to write down solutions to the corresponding Schlesinger system in terms of this differential as well. This solution of the Schlesinger system admits a natural generalization to hyperelliptic curves.