Graduate Student Research Presentations
Exposés de recherche par les étudiants gradués
(Org: **Irena Papst** and/et **Adrien Thierry** (McMaster))

ZAFER SELCUK AYGIN, Carleton University
Fourier series representations of new classes of eta quotients

The Dedekind eta function $\eta(z)$ is the holomorphic function defined on the upper half plane $\{z \in \mathbb{C} \mid \text{Im}(z) > 0\}$ by the product formula

$$\eta(z) = e^{\pi iz/12} \prod_{n=1}^{\infty} (1 - e^{2\pi inz}).$$

We determine Fourier series representations of new classes of etaquotients of weight 2. For example we show that

$$\frac{\eta^3(2z)\eta(4z)\eta^2(8z)}{\eta^2(z)} = \sum_{n=1}^{\infty} \Big(\sum_{m|n} \Big(\frac{8}{m}\Big) m \Big) e^{2\pi inz},$$

where $\left(\frac{8}{m}\right)$ is the Kronecker-Jacobi symbol. We prove our results using the theory of modular forms. This is a joint work with Ayse Alaca and Saban Alaca

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ALI GHASEMI, McMaster University
Simulation of high Reynolds fluid flow through an array of moving cylinders

Because of its practical importance in industrial applications, many attempts have been made to understand the instability of flow-induced vibration of a periodic array of elastically mounted cylinders. It has been suggested that the instability depends on parameters such as geometry, mass damping and natural frequency of the array as well as the mean velocity of the cross flow. Unfortunately, the available experimental provide conflicting evidence for the stabilizing or de-stabilizing role of the flow's Reynolds number and turbulence intensity. In order to investigate this problem, we are using a high resolution pseudo-spectral scheme to solve the Navier-Stocks equations and Brinkman volume penalization to impose no-slip boundary conditions on the surfaces of the moving cylinders. Our goal is to vary the turbulence intensity, Reynolds number and mean velocity to better understand the role of turbulence in fluid-elastic instability. In this presentation I will focus on my initial work developing numerical tools to efficiently simulate the flow on large numbers of processors.

SPENCER HUNT, McMaster University
HPV Type Replacement and Spatial Within-Host Modelling

The human papillomavirus (HPV) is a ubiquitous, multi-type virus that affects many Canadians. Most infections with HPV are transient and cleared by the body naturally. However, persistent infection may lead to pre-cancerous and cancerous lesions. Vaccines against two cancerous types (HPV-16 and -18) have been developed recently. However, researchers are concerned about the possibility of type replacement occurring. This is a phenomenon wherein the vaccination against certain vaccine types of a pathogen causes the niche space for non-vaccine types to be expanded due to the decreased prevalence of these vaccine types. This can result in the increased prevalence of other cancerous non-vaccine HPV types, which may lessen the benefits of the vaccine. Previous mathematical models have been used to examine the potential for type replacement occurring. These models show that HPV type replacement hinges on HPV type interactions. Spatial within-host models can be used to examine HPV types interactions in more detail. This can give researchers and epidemiologists more insight into the potential for HPV type replacement and its implications.

In this talk, I will quickly review some ecological considerations that must be made when developing spatial within-host models for HPV. I will then showcase a spatial model, outlining the important mechanisms of the model. I will wrap up by discussing some analytical results of the model.

SERGEI MELKOUMIAN, McMaster University
The Effects of Wakes and Multiple Moving Bodies on Darwinian Drift

When a body passes through an unbounded fluid, it induces a net displacement of fluid particles. The difference between the initial and final positions of a fluid particle is defined as the particle's Darwinian "drift", and plays an important role in the characterization of the stirring occurring in multiphase flows and due to swimming bodies. For instance, there is evidence to suggest that the drift due to swimming bodies such as zooplankton or larger mammals in the oceans is a significant contributor to overall ocean circulation and nutrient transport. In this talk, we will discuss the effect of vortex wakes on the Lagrangian displacement of particles induced by the passage of an obstacle in a two-dimensional incompressible and inviscid fluid. Further, we will discuss the ongoing work concerning the drift induced by pairs or larger groups of moving obstacles where such flows can be studied using the formalism based on the Schottky-Klein prime function.

ATHINTHRA SETHURAJAN, McMaster University
Reconstruction of State-Dependant Material Properties and Its Application in Electrochemical Systems

Outlining a computational approach to the solution of an inverse modelling problem concerning the material properties of electrolytes used in Lithium-ion batteries is the main theme of this presentation. The dependence of material properties on the concentration of Lithium ions is reconstructed based on the concentration data obtained from an in-situ NMR imaging experiment. This experiment is modelled by a 1D time-dependent PDE describing the evolution of the concentration of Lithium ions with prescribed initial concentration and fluxes at the boundary. The material properties that appear in this model are reconstructed by solving a variational optimization problem in which the least-square error between the experimental and simulated concentration values is minimized. This optimization problem is solved using an innovative gradient-based method in which the gradients are obtained with adjoint analysis. Steps to obtain gradients through adjoint analysis, validational studies on the computational framework for this reconstruction problem and reconstructed material properties of a lab-manufactured and a commercial battery electrolyte are presented with insights which complement available experimental results.

YUSUKE SHIMABUKURO, McMaster University
Nonlinear waves in the integrable systems

Wave phenomena occur everywhere in our surroundings, as for instance in water, acoustic, electromagnetic, and quantum waves, and so on, which have been modeled by nonlinear partial differential equations. Among a plethora of solutions in such equations, a localized and smooth wave retaining its shape, speed, and amplitude is often referred as a soliton. We will present orbital stability of a soliton in the integrable system on a class of nonlinear Dirac equations. This is a joint work with Dmitry Pelinovsky and Andres Contreras.

PETER SINCLAIR, McMaster University
Computable Axiomatizability of Elementary Classes

Determining whether a class of model-theoretic structures has a computable axiomatization is a straightforward question which can have surprising answers. The class of all fields, for example, is easily axiomatized, as is the class of fields of characteristic zero; however, the class of all fields of non-zero characteristic is not.

A more complicated example is the class consisting of ultraproducts of o-minimal structures (definitions of these terms will be given during the presentation). Multiple proposals were made for possible axiomatizations of this class, but Alex Rennet showed in a recent paper that in any proper expansion of the ordered ring language, this class is not axiomatizable. I will present a generalized version of Rennet's theorem, along with examples of classes that the theorem can be applied to.

ADRIEN THIERRY, McMaster
Inversion factor.

In combinatorics on words, counting the different types of squares is a fundamental problem. It was well understood that at most two different squares could have their last occurrence starting at the same position, but little was known about them. The recent advance in the field relies on the combinatorial structures of those double-squares. We will take a look here at those structures, and a particularly important one: the inversion factor, and see how they can lead to a better bound for the maximal number of distinct squares in a string.