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First-order system Petrov-Galerkin discretization for a singularly perturbed reaction-diffusion problem

This talk presents a Petrov-Galerkin finite-element discretization of a singularly perturbed reaction-diffusion equation posed on the unit square. We extend the work of Lin and Stynes (2012), who suggest that the natural energy norm (associated with a standard Galerkin approach) is not an appropriate setting for analyzing such problems. They propose a method for which the natural norm is "balanced", reflecting important features of the continuum solution, but which requires discretization in an $H(\text{div})$ space. Here, in the style of a first-order system least squares (FOSLS) method, we extend their approach by introducing an additional constraint that simplifies the associated finite-element space and the resulting analysis. We prove robust convergence in a balanced norm on a mesh with a priori adaptation, presenting supporting numerical results and demonstrating optimal solution of the resulting linear systems using multigrid methods.