## Dynamical Systems with Applications in Mathematical Biology
## Systèmes dynamiques et applications en biologie mathématique
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**JACQUES BELAIR**, Université de Montréal
*An age-structured model of erythropoiesis with dynamical destruction rate*

We consider an age-structured population model of erythropoiesis including a dynamical death rate for mature cells as an attempt to understand erythrocyte lifespan distribution, a feature which may have clinical significance. The model takes the form of a system of nonlinear delay differential equations representing total circulating cells, the regulatory hormone (erythropoietin) and the moving maximal lifespan of mature cells. Linear stability of the non trivial steady states yields an intricate characteristic equation, as well as a stability chart displaying the boundary of the region of stability. Numerical simulations strongly suggest a supercritical Hopf bifurcation, and mode interactions.

Joint work with Frédéric Paquin-Lefebvre

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**FRED BRAUER**, University of British Columbia
*Some simple nosocomial disease transmission models*

The SARS epidemic of 2002-3 drew attention to nosocomial disease transmission as many of the disease cases were transmitted through hospital staff and visitors. Various types of model have been proposed to describe this, including metapopulation models. We formulate and analyze a simple compartmental model with heterogeneous mixing to describe nosocomial transmission and determine the reproduction number and final size relation.

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**PIETRO-LUCIANO BUONO**, University of Ontario Institute of Technology
*Symmetry and Bifurcations in First-order Hyperbolic PDEs with Nonlocal Terms Modelling Animal Aggregation*

Pattern formation in self-organised biological aggregation is a phenomenon that has been studied intensively over the past twenty years. I will present a class of models for animal aggregation in the form of two first-order hyperbolic partial differential equations on a one-dimensional domain with periodic boundary conditions, describing the motion of left and right moving individuals with nonlinear, nonlocal social interaction terms for attraction, repulsion and alignment giving the turning rates of individuals. This class of models has been introduced in the Ph.D thesis of R. Eftimie. In this talk, I will show that the equations are O(2)-equivariant where the group O(2) is generated by space-translations and a reflection which interchanges left-moving individuals with right-moving individuals across the middle of the interval. I will discuss steady-states and their symmetry with a focus on homogeneous O(2) symmetric states and the existence of codimension two steady-state/steady-state, steady-state/Hopf and Hopf/Hopf bifurcation points. I will present how using existing symmetry-breaking bifurcation theory and new theoretical results, one can study the neighborhood of those bifurcation points and classify the patterns obtained. This is joint work with R. Eftimie (U. Dundee, Scotland).

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**SUE ANN CAMPBELL**, University of Waterloo
*Mean Field Analysis of Networks of Neurons*

We use mean field analysis to study the behaviour of networks of all-to-all pulse-coupled neurons. The individual neurons are represented using a class of two-dimensional integrate and fire model. The mean field model is a system of switching ordinary differential equations, which can undergo both standard and nonsmooth bifurcations. The results of the mean field analysis are compared with numerical simulations of large networks. This is joint work with Wilten Nicola.

**YUMING CHEN**, Wilfrid Laurier University
*The global stability of an SIRS model with infection age*

Infection age is an important factor affecting the transmission of infectious diseases. In this talk, we consider an SIRS model with infection age, which is described by a mixed system of ordinary differential equations and partial differential equations. The expression of the basic reproduction number $R_0$ is obtained. If $R_0 \leq 1$ then the model only has the disease-free equilibrium, while if $R_0 > 1$ then besides the disease-free equilibrium the model also has an endemic equilibrium. Moreover, if $R_0 < 1$ then the disease-free equilibrium is globally asymptotically stable otherwise it is unstable; if $R_0 > 1$ then the endemic equilibrium is globally asymptotically stable under additional conditions. The theoretical results are illustrated with numerical simulations. This is a joint work with J. Yang and F. Zhang.

**GUIHONG FAN**, Columbus State University (Georgia State)
*The Bifurcation Study of 1:2 Resonance in a Delayed System of Two Coupled Neurons*

In this paper, we consider a delayed system of differential equations modeling two neurons: one is excitatory, the other is inhibitory. We study the stability and bifurcations of the trivial equilibrium. Using center manifold theory for delay differential equations, we develop the universal unfolding of the system when the trivial equilibrium point has a double zero eigenvalue. In particular, we show a universal unfolding may be obtained by perturbing any two of the parameters in the system. Our study shows that the dynamics on the center manifold are characterized by a planar system whose vector field has the property of 1:2 resonance, also frequently referred as the Bogdanov–Takens bifurcation with Z2 symmetry. We show that the unfolding of the singularity exhibits Hopf bifurcation, pitchfork bifurcation, homoclinic bifurcation, and fold bifurcation of limit cycles. The symmetry gives rise to a “figure-eight” homoclinic orbit.

This is a joint work with Sue Ann Campbell (University of Waterloo), Gail S. K. Wolkowicz (McMaster University), and Huaiping Zhu (York University).

**HONGBIN GUO**, Public Health Agency of Canada
*Immigration Disease Models*

With immigration and emigration more common across the globe, disease prevention and control associated with migration poses a great challenge for public health authorities among migrant-receiving countries. This presentation reviews immigration-related modelling works from mathematical and epidemiological literatures and shows recent progress in heterogeneity modelling and new issues.

**TONY HUMPHRIES**, McGill University
*Modelling myelopoiesis with state-dependent delay differential equations*

The production of white blood cells is modelled from hematopoietic stem cells (HSCs) through proliferating and maturing precursors to circulating neutrophils. The main cytokine that regulates this process is Granulocyte Colony Stimulating Factor (G-CSF). G-CSF regulates the differentiation rate of HSCs, the proliferation rate during mitosis, the maturation time, and the rate at which mature neutrophils are released into circulation from the bone marrow. We model the variable maturation time via an age-structured PDE model with variable ageing rate which results in a system of state-dependent delay differential equations. We also describe a new model of the G-CSF pharmacokinetics. Determining the parameters is another difficulty, but we show that the model derivation implies associated constraints which we apply in determining the parameters.

**SILVANA ILIE**, Ryerson University
*Effective methods for stochastic biochemical kinetics*

Stochastic modelling is essential for studying key biological processes, such as signaling chemical pathways in a cell, when some molecular species are in low numbers. The random fluctuations due to low amounts of certain biochemically reacting species have been observed experimentally. Mathematically, the dynamics of these biochemical systems may be accurately described using Markov processes. The biochemical systems arising in applications often evolve on multiple interacting time-scales, meaning their mathematical models are stiff. In this talk I shall present effective methods for overcoming stiffness in stochastic simulations of biochemical systems. We consider spatially homogeneous and heterogeneous models of biochemical kinetics.

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**BILL LANGFORD**, University of Guelph
*Huygens' clocks revisited*

The first reported observation of synchronization of coupled oscillators was by Christiaan Huygens in 1665. He observed that, if two of his clocks were weakly coupled, after a short time they synchronized with opposite displacements and velocities, i.e. "anti-phase synchronization". Huygens was not able to explain his observation and it has been a topic of study to this day.

Our contributions to this study are as follows. First, we observed that Huygens' clocks were identical and symmetrically coupled; that is, they had a $(\mathbb{Z}_2)$ permutation symmetry. Second, we observed that Huygens had reduced the linear friction of each clock-oscillator to nearly zero; that is, each was close to a Hopf bifurcation. Then we carried out a general analysis of double Hopf bifurcation with Huygens symmetry, using equivariant normal forms. This study revealed a rich variety of dynamic behaviours, including both in-phase and anti-phase normal modes and pairs of mixed-mode phase-locked periodic solutions. A theorem based on topological degree theory establishes the existence of quasiperiodic solutions in an invariant 3-torus that resembles a 2-torus "toroidal breather". An Arnold tongue plays a fundamental role in the secondary bifurcations to either phase-locked periodic solutions or quasiperiodic solutions. Numerical analysis using Matlab extends the local bifurcation analysis to a more global picture. Finally, application of this general theory to Huygens' clocks predicts his observation of "anti-phase synchronization".

This is joint work with P. M. Kitanov and A. R. Willms.

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**VICTOR LEBLANC**, University of Ottawa
*Lattice symmetry breaking perturbations for spiral waves*

Spiral waves in two-dimensional excitable media have been observed experimentally and studied extensively. It is now well-known that the symmetry properties of the medium of propagation drives many of the dynamics and bifurcations which are experimentally observed for these waves. Also, symmetry-breaking induced by boundaries, inhomogeneities and anisotropy have all been shown to lead to different dynamical regimes as to that which is predicted for mathematical models which assume infinite homogeneous and isotropic planar geometry. Recent mathematical analyses incorpo- rating the concept of forced symmetry-breaking from the Euclidean group of all planar translations and rotations have given model-independent descriptions of the effects of media imperfections on spiral wave dynamics. In this talk, we continue this program by considering rotating waves in dynamical systems which are small perturbations of a Euclidean-equivariant dynamical system, but for which the perturbation preserves only the symmetry of a regular square lattice. This is joint work with Laurent Charette.

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**MICHAEL LI**, University of Alberta
*Developing a Mathematical Modeling Approach to Public Health Assessment: HIV in China as a Case Study*

The HIV infection can have a long incubation period before symptoms appear. The current HIV pandemic has produced a large population of HIV positive people who are not diagnosed or unaware of their HIV status, while capable of transmitting the infection. It was estimated that close to a quarter of the 70,000 people living with HIV in Canada at the end of 2011 were undiagnosed. Estimation of the size of undiagnosed HIV positive population is an annual/bi-annual exercise for public health agencies in all developed countries and the WHO, simply because this population is an obstacle to HIV control and prevention. From the viewpoint of public health assessment, the presence of an undiagnosed population makes it difficult to estimate the true incidence, prevalence, and mortality of the HIV epidemic, and to assess the effectiveness of control and preventive measures. Estimation of undiagnosed population using HIV surveillance data has been done using a statistical methodology called (modified) back-calculation. In this talk, I will present our work on developing a rigorous mathematical modeling approach to this problem, and our collaboration with China CDC on the assessment of the HIV epidemic in China.

I will show how statistical methodologies are integrated with differential equation models, and point out several important modeling issues that tend to be overlooked. Our objective is to develop mathematical modeling, integrated with statistical methodologies and health data, as an effective tool for public health assessment.

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**XINZHI LIU**, University of Waterloo

*A generalized invariance principle for infectious disease models with switching and pulse control*

Mathematical models for infectious disease are crucial in gaining knowledge of the underlying mechanism that drives an epidemic. They are often used for implementing and evaluating control schemes in order to eradicate a disease. This talk discusses some epidemic models with switching parameters and pulse control. Hybrid control schemes are examined, and, in doing so, we hope to gain insight into the effects of a time-varying contact rate on critical control levels required for eradication. By introducing the notions of persistent limit set and persistent mode, we extend the classical LaSalle's invariance principle to epidemic models with switching parameters and pulse control. A weak invariance principle is established for such systems, under a weak dwell-time condition on the impulsive and switching signals. This weak invariance principle is then applied to establish sufficient conditions for the global asymptotic stability of the disease-free solution, which may give some insight into the effects of a time-varying contact rate on critical control levels required for eradication of a disease.

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**FELICIA MAGPANTAY**, University of Michigan

*Fitting models of imperfect vaccines to pertussis incidence data*

The resurgence of pertussis in some countries with high vaccine coverage (e.g. Canada, USA, UK) has highlighted how much we still do not know about the dynamics of this disease, particularly the type of immunity rendered by infection and vaccination. There has been much discussion on the different modes by which vaccines might fail. When a vaccine reduces the probability of infection upon exposure but does not eliminate it, this is called failure in *degree*. When the protection conferred wanes over time, this is termed failure in *duration*. Vaccines may also protect against disease but not infection and transmission. Towards the goal of determining the key features of the pertussis vaccine, we fitted an assortment of imperfect vaccine models to pertussis incidence data. Trajectory matching and Maximization via iterated Filtering (MIF) was used to obtain the maximum likelihood estimates of the model parameters using different observation models. The idea behind these algorithms and the results of its application to pertussis data will be presented.

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**CONNELL MCCLUSKEY**, Wilfrid Laurier University

*Disease Models with Immigration*

This talk will include recent and current work on several models of infectious disease spread that include immigration of individuals who are infected with the disease. Both finite dimensional (ODE) and infinite dimensional (age-structured PDE) models are considered.

This work is in collaboration with students Ram Sigdel and Sarah Henshaw.

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**GERGELY ROST**, Bolyai Institute, University of Szeged, Hungary

*Endemic Bubbles Generated by Delayed Behavioral Response in Epidemic Models*

Several models have been proposed to capture the phenomenon that individuals modify their behavior during an epidemic outbreak. This can be due to directly experiencing the rising number of infections, media coverage, or intervention policies. In this talk we show that a delayed activation of such a response can lead to very interesting dynamics even in simple epidemic settings. For SIS type process, if the delayed response is relatively weak, the system preserves global stability, regardless of the delay. However, for stronger responses, we can observe stability switches as the basic reproduction number is increasing. First, the stability is passed from the disease free equilibrium to an endemic equilibrium via transcritical bifurcation as usual, but a further increase of the reproduction number causes sustained oscillations, which later disappear for high reproduction numbers, forming a structure in the bifurcation diagram what we call endemic bubble. Detailed mathematical analysis is given for the typical media functions used in the literature.

Joint work with Maoxing Liu, Eduardo Liz, Gabriella Vas.

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**ROBERT SMITH?**, The University of Ottawa
*Can mathematics change the world? Insights into policy changes using HPV modelling as an example*

We use a mathematical model to investigate the influence of quadrivalent vaccine programs, which differ by the grade of vaccination and number of doses given, on the prevalence of HPV 6, 11, 16 and 18. We address the following research questions: 1. Does the grade at which the girls are vaccinated significantly affect the outcome of the program? 2. What coverage rate must the provinces reach in order to reduce the impact of HPV on the Canadian population? 3. What are the implications of vaccinating with two vs. three doses? In addition, we offer insights into how mathematical models can make their way into policy and contribute to real-world changes, such as the Quebec policy change of the recommended HPV doses.

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**PETER STECHLINSKI**, University of Waterloo
*Control strategies for a seasonal model of chikungunya disease*

We present a model for the outbreak of chikungunya, a vector-borne disease, on Reunion island in 2005-06. Seasonality is added to the model by considering variations in the birth rate of the mosquito population and variations in the contact rate between mosquitoes and humans. Possible control strategies are presented and studied theoretically (for example, the mechanical destruction of mosquito breeding sites). The control schemes are compared numerically by calculating an efficacy rating under different scenarios (for example, varying the duration of the scheme).

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**LIN WANG**, University of New Brunswick
*Biological systems with multiple stable states*

In this talk, I will present some of our recent work on biological systems with multiple stable states. We show that these models could exhibit multistability phenomena: (i) node-node bistability; (ii) node-cycle bistability; (iii) cycle-cycle bistability; (iv) node-node-cycle tristability and (v) node-node-torus tristability. The talk is based on joint work with Xi Hu, Hongying Shu and James Watmough.

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**MARION WEEDERMANN**, Dominican University
*Optimization of biogas production in a model for anaerobic digestion*

We investigate a model for anaerobic digestion, a process used to produce biogas. The model describes the interactions of microbial populations involved in three main stages of anaerobic digestion: acidogenesis, acetogenesis, and methanogenesis. We show that an increased yield in biogas was achieved in regions where operating parameters pushed the system into a bistable state. In some regions of bistability, biogas production occurred at only one of the steady states while in others both steady-states resulted in biogas production with one state being more productive than the other. We demonstrate which operating parameters and state variables had the most significant impact on system performance. Surprisingly, the optimal biogas production did not always occur at a steady state where all classes of microorganisms coexist. This is joint work with Gail Wolkowicz.

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**JIANHONG WU**, York University, York Institute for Health Research
*Mathematics for food-borne disease spread: water chemistry and pathogen cross-contamination dynamics during washing procedures*

We use epidemic models to describe water chemistry and pathogen cross-contamination dynamics in a chemostat-like wash tank for fresh produce. Our model templates involve periodic coefficients and circulation delays, and models are parametrized using data from a pilot plant scale evaluation of the US Department of Agriculture (UDA). Implication of model dynamics for industrial guidelines will be discussed. This is based on an on-going collaboration with D. Munther and P. Srinivasan

(Cleveland state), X. Sun, S. Tang, Y. Xiao (Xian Jiaotong), F. Magpantay (Michaigan), H. Shimozako (Sao Paulo), Yaguang Luo (USDA), A. Fazil and B. Smith (Public Health Agency of Canada).

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**XIAOQIANG ZHAO**, Memorial University of Newfoundland
*Propagation Phenomena for A Reaction and Diffusion Model with Seasonal Succession*

In this talk, I will report our recent research on propagation phenomena for a two-species competitive reaction and diffusion model with seasonal succession. In the bistable case, we establish the existence and global stability of time-periodic bistable travelling waves. In the monostable case, we show that the model system admits a single spreading speed, which coincides with the minimal wave speed for time-periodic traveling waves. We also obtain a set of sufficient conditions for the spreading speed to be linearly determinate. This talk is based on two joint papers with Drs. Yuxiang Zhang and Manjun Ma, respectively.

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**HUAIPING ZHU**, York
*Bifurcation of limit cycles in predator-prey models, canard cycles and Hilbert's 16th problem*

In this talk, I will start with predator-prey system to present the limit cycles and their bifurcations, including canard cycles and their cyclicity as well as fast-slow dynamics. I will then use the predator-prey system to reveal the two types of mechanisms of fast-slow dynamics in mathematical biology. In the end, I will connect the finiteness part of Hilbert's 16th problem, to explain the difficulties in dealing with the finite cyclicity of a degenerate degenerate graphics, the last challenge towards the proof of the finiteness part of Hilbert's 16th problem.

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**XINGFU ZOU**, University of Western Ontario
*Budding vs lysis—- on viral release strategies in virus replication*

We consider the evolutionary competition between budding and lytic viral release strategies, using a delay differential equation model with distributed delay. The role of antibody is incorporated. Analysis of the model predicts under what conditions the budding strategy is superior to the lytic one and in what situations the lytic strategy will be advantageous over the budding one. This is a joint work with Dr. Xiulan Lai