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Algebraic cycles and Siegel modular forms

In this lecture I will discuss the role played by modular generating series for the classes defined by certain algebraic cycles on a class of locally symmetric varieties.

The theory of divisors modulo the divisors of rational functions on a smooth complex projective variety X is well understood. For example, for a smooth projective curve defined over a number field, the divisor classes of degree zero correspond to rational points on the Jacobian of X and this group is finitely generated by the Mordell-Weil Theorem. For algebraic cycles of higher codimension, our understanding is much more limited. For example, for a variety defined over a number field K, the Chow group of cycles of codimension r, $r > 1$, defined over K modulo rational equivalence is not known to be finitely generated.

The locally symmetric varieties X associated to rational quadratic forms of signature $(n, 2)$ are particularly nice test cases since they have explicitly constructed families of algebraic cycles of all codimensions. For a fixed codimension r, one can form a generating series for the classes of such special cycles. I will discuss (1) What information is contained in the statement that such a generating series is a modular form? (2) Borcherds' proof of modularity in the case of divisors. (3) Recent results of Wei Zhang, Martin Raum and Jan Bruinier for higher codimension.