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The Four-Dimensional Perfect Mirsky Conjecture

A square matrix with non-negative entries, all of whose rows and columns sum to 1 is called a doubly stochastic matrix. The set of such matrices of size $n \times n$ is denoted Ω_n. Doubly stochastic matrices are closely tied to majorization, a partial order on vectors in $\mathbb{R}^n$, a connection made explicit by the Hardy-Littlewood-Polya theorem. Majorization plays an important role in quantum information, where it can be used to compare entanglement of two quantum states. In 1965, Perfect and Mirsky conjectured that the region of all possible eigenvalues of all $n \times n$ doubly stochastic matrices (denoted ω_n) would be the union of the regions Π_k for $k \in \{1, 2, \cdots, n\}$, where Π_k is the convex hull of the k^{th} roots of unity. They proved the conjecture for $n = 1, 2, 3$. In 2007, Rivard and Mashreghi exhibited a counterexample for $n = 5$. We prove the Perfect-Mirsky conjecture for $n = 4$, and provide a new conjecture for which Rivard and Mashreghi's example is not a counterexample. We also discuss some geometric interpretations of the problem of characterizing ω_n.