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Torsion points and the Lattes family

We give a dynamical proof of a result of Masser and Zannier: there are only finitely many parameters $t \in \mathbb{C}$ for which points $P_t = (2, \sqrt{2(2-t)})$ and $Q_t = (3, \sqrt{6(3-t)})$ are both torsion on the Legendre elliptic curve $E_t = \{y^2 = x(x-1)(x-t)\}$. A key ingredient in the proof is the arithmetic equidistribution theorem on $\mathbb{P}^1$ of Baker-Rumely, Favre- Rivera-Letelier and Chambert-Loir, applied to parameters $t \in \bar{\mathbb{Q}}$ for which a given point $a \in \bar{\mathbb{Q}}$ is preperiodic for the degree-4 Lattes family $f_t : \mathbb{P}^1 \to \mathbb{P}^1$. Our main new results involve complex dynamics and potential theory: (1) for each $c \in \mathbb{C}(t)$, the bifurcation measure μ_c for the pair (f_t, c) has continuous potential across the singular parameters $t = 0, 1, \infty$; and (2) for distinct points $a, b \in \mathbb{C} \setminus \{0, 1\}$, the bifurcation measures μ_a and μ_b cannot coincide. We also compute the homogeneous capacity of the bifurcation set for each marked point $c \in \mathbb{C}(t)$.