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Scaling ratios for Siegel disks

The boundary of the Siegel disk of a quadratic polynomial with an irrationally indifferent fixed point and the rotation number whose continued fraction expansion is preperiodic has been observed to be self-similar with a certain scaling ratio. The restriction of the dynamics of the quadratic polynomial to the boundary of the Siegel disk is known to be quasisymmetrically conjugate to the rigid rotation with the same rotation number. The geometry of this self-similarity is universal for a large class of holomorphic maps.

We describe an estimate on the quasisymmetric constant of the conjugacy, and use it to prove bounds on the scaling ratio λ of the form

$$\alpha^{\gamma} \le |\lambda| \le \delta^s,$$

where s is the period of the continued fraction, and $\alpha \in (0,1)$ depends on the rotation number in an explicit way, while δ , $\gamma \in (0,1)$ depend only on the maximum of the integers in the continued fraction expansion of the rotation number.