PEI YU, Western University Computation of normal forms of delay differential equations

Computing normal forms of differential equations (DE) is important in the study of dynamical behaviors such as stability and bifurcation, which is particularly difficult and tedious for delay differential equations (DDE). The center manifold reduction (CMR) method is usually applied to compute normal forms of DDEs. Recently, the multiple time scales (MTS) method has been directly applied to compute normal forms of DDEs and shown to be simpler than the CMR method. In this talk, we will show the equivalence of the MTS method and the CMR method in computing the normal forms of DDEs. The delays involved in the DDEs can be discrete or distributed. Particular attention is focused on dynamics associated with the semisimple singularity. The two methods are proved to be equivalent for ordinary differential equations up to any order, while up to third order for DDEs under certain conditions, which can be satisifed in most of practical problems. The equivalence of the two methods can be extended to other types of DDEs, including neutral functional differential equations (NFDE) (or neutral delay differential equations (NFDE)), and partial functional differential equations (PFDE). Several practical systems with delays are presented to demonstrate the application of the methods, associated with Hopf, Hopf-zero, and double-Hopf singularities.