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Koszul Algebras associated to zonotopes and CAT(0) cube complexes

Motivated by the study of Markov chains associated to random walks on chambers of hyperplane arrangements, Bidigare, Hanlon and Rockmore introduced the study of the algebra of a hyperplane face semigroup. This work was continued by Brown and Diaconis and then by Saliola.

Hyperplane arrangements are dual to polytopes called zonotopes and so we argue these really should be thought of as zonotope face semigroups. It turns out that there are many other regular cell complexes, including zonotopes, oriented matroids, complex oriented matroids and CAT(0) cube complexes, whose face posets admit a semigroup structure analogous to that of a hyperplane face semigroup.

The algebras of these semigroups over a field turn out to be unital and have the same quiver presentation over any field, which we compute explicitly They turn out to be graded by path length and Koszul. Their Koszul duals are incidence algebras of certain Cohen-Macaulay posets. The minimal projective resolutions of the simple modules can be interpreted as the augemented cellular chain complexes of related cell complexes.