JAIMAL THIND, University of Toronto - Mississauga

Reconstruction of Lie Group Morphisms

Given a map of Lie groups $f: G \to H$, one gets a functor $f^*: Rep(H) \to Rep(G)$ pulling back a representation of H to G via the map f. This functor induces a map of semirings $f^*: \mathcal{K}^+(H) \to \mathcal{K}^+(G)$, where $\mathcal{K}^+$ denotes the semiring of isomorphism classes of representations, with direct sum and tensor product as addition and multiplication. It has been shown by Kazhdan, Larsen, and Varshavsky that when G is reductive, the semiring $\mathcal{K}^+(G)$ determines G up to isomorphism. Moreover, they show that if a map of semirings preserves irreducibles, then it comes from a map of Lie Groups. They also showed that not all maps of semirings $\mathcal{K}^+(H) \to \mathcal{K}^+(G)$ come from maps of Lie groups $G \to H$. However, their examples do not preserve dimension, and few maps of Lie Groups preserve irreducibles.

A natural question then is to classify which maps of semirings do come from maps of Lie groups. We will discuss this problem and report on recent progress made for the case of simple Lie groups. In particular, when H is classical, it suffices to require that the map of semirings preserves the structure on $\mathcal{K}^+$ coming from the exterior power operation on representations. This is based on joint work in progress with J. Blasiak and J. Grochow.