Probability Theory and Mathematical Physics
Méthodes probabilistiques et physique mathématique
(Org: **Louigi Addario-Berry** (McGill) and/et **Louis-Pierre Arguin** (Montréal))

OMER ANGEL, UBC
Half planar maps

We characterise all measures on half planar maps that satisfy a domain Markov property, and discuss some of their geometric properties. Joint work with Gourab Ray.

ALEX BLOEMENDAL, Harvard University
Spiked random matrices

Spiked models are finite rank perturbations of large Wigner or sample covariance matrices; they have received a fair bit of attention since the discovery eight years ago of a certain phase transition phenomenon. The phase transition is in the behaviour of the largest eigenvalues and corresponding eigenvectors as a function of the perturbation. I will describe various ways to approach and understand this phenomenon and survey some results, including joint work with B. Virág as well as with A. Knowles, H.-T. Yau and J. Yin.

PAUL BOURGADE, Harvard University
Universality for beta ensembles

Wigner stated the general hypothesis that the distribution of eigenvalue spacings of large complicated quantum systems is universal in the sense that it depends only on the symmetry class of the physical system but not on other detailed structures. The simplest case for this hypothesis concerns large but finite dimensional matrices. Spectacular progress was done in the past two decades to prove universality of random matrices presenting an orthogonal, unitary or symplectic invariance. These models correspond to log-gases with respective inverse temperature 1, 2 or 4. I will report on a joint work with L. Erdős and H.-T. Yau, which yields universality for the log-gases at arbitrary temperature, for analytic external potential, at the microscopic scale.

DAVID GAMARNIK, MIT
Convergent sequences of sparse graphs: A large deviations approach

The theory of converging graph sequences is well developed for the class of dense graphs. The theory of converging sparse graph sequences, however, is far less understood. We show that prior definitions of converging sparse graph sequences are inadequate to capture important graph theoretic and statistical physics properties, and introduce a new definition based on the large deviations theory. We show that the new definition implies most the known types of convergences and conjecture that sparse random graphs are converging in the sense of the new definition. Establishing this conjecture will have an important implications for the theory of spin glasses.

Joint work with Jennifer Chayes and Christian Borgs.

MICHAEL KOZDRON, University of Regina
Bayes' rule for quantum random variables and positive operator valued measures

A quantum probability measure is a function on a sigma-algebra of subsets of a (locally compact and Hausdorff) sample space that satisfies the formal requirements for a measure, but whose values are positive operators acting on a complex Hilbert space, and a quantum random variable is a measurable operator valued function. In this talk, we introduce a quantum analogue for the expected value of a quantum random variable relative to a quantum probability measure. We also introduce a quantum conditional expectation which results in quantum versions of some standard identities for Radon-Nikodym derivatives as well as a quantum analogue of Bayes' rule. This talk is based on joint work with Doug Farenick of the University of Regina.

ANNA LEVIT, University of British Columbia
Ground states for mean field models with a transverse component

We investigate global logarithmic asymptotics of ground states for a family of quantum mean field models. Our approach is based on a stochastic representation and a combination of large deviation and weak KAM techniques. The spin- 1/2 case is worked out in more detail.

DANA MENDELSON, MIT
Rate of Convergence for Cardy's Formula

We consider 2D critical site percolation on the triangular lattice in a piecewise analytic Jordan domain. In this talk, I will show that crossing probabilities for such domains converge with power law rate in the mesh size to their limit given by the Cardy-Smirnov formula. I will also show how this result can be used to obtain new upper and lower bounds of $e^{O(\sqrt{\log\log R})}R^{-1/3}$ for the probability that the cluster at the origin in the half-plane has diameter R, improving the previously known estimate of $R^{(-1/3+o(1))}$.

This is joint work with Asaf Nachmias and Samuel S. Watson.

JASON MILLER, MIT
Imaginary Geometry and the Gaussian Free Field

The Schramm-Loewner evolution (SLE) is the canonical model of a non-crossing conformally invariant random curve, introduced by Oded Schramm in 1999 as a candidate for the scaling limit of loop erased random walk and the interfaces in critical percolation. The development of SLE has been one of the most exciting areas in probability theory over the last decade because Schramm's curves have now been shown to arise as the scaling limit of the interfaces of a number of different discrete models from statistical physics. In this talk, I will describe how SLE curves can be realized as the flow lines of a random vector field generated by the Gaussian free field, the two-time-dimensional analog of Brownian motion, and how this perspective can be used to study the sample path behavior of SLE. Based on joint works with Scott Sheffield.

ASAF NACHMIAS, University of British Columiba
Hypercube percolation

Consider percolation on the Hamming cube $\{0,1\}^n$ at the critical probability p_c (at which the expected cluster size is $2^{n/3}$). It is known that if $p = p_c(1+O(2^{-n/3})$, then the largest component is of size roughly $2^{2n/3}$ with high probability and that this quantity is non-concentrated. We show that for any sequence $\epsilon(n)$ such that $\epsilon(n) \gg 2^{-n/3}$ and $\epsilon(n) = o(1)$ percolation at $p_c(1+\epsilon(n))$ yields a largest cluster of size $(2+o(1))\epsilon(n)2^n$.

This result settles a conjecture of Borgs, Chayes, van der Hofstad, Slade and Spencer.

Joint work with Remco van der Hofstad.

JEREMY QUASTEL, University of Toronto
Regularity and variational problems for Airy processes

The Airy processes are stochastic processes that have come out of both random growth models and random matrix theory. They are defined in terms of their finite dimensional distributions which are given by large Fredholm determinants. However, this description is not so useful for proving local path properties or for solving variational problems which arise in a natural way. We give an alternate description and show how it can be used to obtain such information.

MATT ROBERTS, University of Warwick
Intermittency in branching random walk in random environment

Over the last 20 years mathematicians have proved rigorously that the parabolic Anderson model shows the intermittency behaviour predicted by physicists. We shall see that a branching random walk in Pareto random environment displays the same qualitative behaviour, but with several important differences.

BALINT VIRAG, University of Toronto
The sound of random graphs

Infinite random graphs, such as Galton-Watson trees and percolation clusters may have real numbers that are eigenvalues with probability one, providing a consistent "sound". These numbers correspond to atoms in their density-of-states measure.

When does the sound exist? When are there only finitely many atoms? When is the measure purely atomic? I will review many examples and show some elementary techniques for studying these problems, including some developed in joint works with Charles Bordenave and Arnab Sen. The last question is open for percolation clusters in Z^d, $d \geq 3$, and for incipient Galton-Watson trees.