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"The Foundations of Probability and Statistics with Black Swans"

Classical mechanics, probability and statistics neglect rare events. The associated stochastic systems represent smooth transition while real systems exhibit jumps as well as continuous evolution. The universe is bumpy as well as continuous and so are most of natural systems, such as earth shifts, as well as social systems such as the economy and financial markets. We present a new axiomatic treatment of probability and statistics with black swans, which are rare events with momentous consequences. The new axioms differ from traditional axioms of probability and statistics in that we require 'sensitivity to rare events'. A representation theorem identifies a new type of measures on R that has both countably additive and purely finitely additive parts. This leads to distributions with heavy tails, and to stochastic systems that result in jump - diffusion processes through time. The new axioms are compared with the standard axioms of mathematics and probability theory and are shown to differ in a crucial axiom ("Monotone Continuity, S.P.4.") that is generally invoked and is restrictive enough to eliminate heavy tails and to underestimate rare events. The new theory is able to integrate ambiguous features of mathematics and includes aspects of Godel's Incompleteness Theorem as well as the Independence of the Continuum Hypothesis and the Axiom of Choice, and is applied to practical examples of measure theory, probability and stochastic systems in natural systems as well as in financial markets.