GEORGE ELLIOTT, University of Toronto

What does classification mean?

From the point of view of C*-algebras, one should perhaps agree that classification means determining in some useful way the approximate unitary equivalence class of a morphism—as these form a category, which for separable C*-algebras distinguishes isomorphism classes. (An analogous situation holds for countable discrete groups.) For interesting classes of separable (amenable) C*-algebras, this has been achieved, by means of K-theoretical invariants. (On the other hand, no such generic invariants seem to work for countable groups.) A way of testing how difficult various cases are—the especially well behaved amenable case, in the sense of Toms and Winter, the general amenable case, or the general separable case—might be to look at the Borel complexity of the isomorphism relation (with respect to a natural Borel structure), in the sense of descriptive set theory. As it happens (work of many hands), except for the special case of AF algebras (dealt with by Bratteli and me forty years ago), which is simpler than the other cases, there would seem to be no noticeable variation at all in the Borel complexity (among all three of the cases mentioned above).