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A Variant of Lehmers Conjecture in the CM Case

Lehmer's conjecture asserts that $\tau(p) \neq 0$, where τ is the Ramanujan τ-function. This is equivalent to the assertion that $\tau(n) \neq 0$ for any n. A related problem is to find the distribution of primes p for which $\tau(p) \equiv 0 \pmod{p}$. These are open problems. However, the variant of estimating the number of integers n for which n and $\tau(n)$ do not have a non-trivial common factor is more amenable to study. More generally, let f be a normalized eigenform for the Hecke operators of weight $k \geq 2$ and having rational integer Fourier coefficients $\{a(n)\}$. It is interesting to study the quantity $(n, a(n))$. It was proved by S. Gun and V. K. Murty (2009) that for Hecke eigenforms f of weight 2 with CM and integer coefficients $a(n)$

$$\{n \leq x \mid (n, a(n)) = 1\} = \frac{(1+o(1))U_f x}{\sqrt{\log x \log\log\log x}}$$

for some constant U_f. We extend this result to higher weight forms.
We also show that

$$\{n \leq x \mid (n, a(n)) \text{ is a prime}\} \ll \frac{x \log\log\log\log x}{\sqrt{\log x \log\log\log x}}.$$