JIM PARKS, Concordia University

An upper bound for the average number of amicable pairs

Let E be an elliptic curve over $\mathbb{Q}$. Silverman and Stange defined a pair (p,q) of rational primes to be an *amicable pair* for E if E has good reduction at these primes and the number of points on the reductions $\widetilde{E}_p$ and $\widetilde{E}_q$ satisfy $\#\widetilde{E}_p(\mathbb{F}_p)=q$ and $\#\widetilde{E}_q(\mathbb{F}_q)=p$. Let $Q_E(X)$ denote the number of amicable pairs (p,q) for $E/\mathbb{Q}$ with $p\leq X$. They conjectured that $Q_E(X)\asymp X/(\log X)^2$ if E does not have complex multiplication. This conjecture was refined by Jones by specifying the appropriate constants. In this talk I will show that the conjectured upper bound holds for $Q_E(X)$ on average over the family of all elliptic curves.