MIKE ROTH, Queen's University

Seshadri constants, diophantine approximation, and Roth's theorem for arbitrary varieties

If X is a variety of general type defined over a number field k, then the Bombieri-Lang conjecture predicts that the k-rational points of X are not Zariski dense. One way to view the conjecture is that a global condition on the canonical bundle (that it is "generically positive") implies a global condition about rational points. By a well-established principle in geometry we should also look for local influence of positivity on the accumulation of rational points. To do that we need measures of both these local phenomena.

Let L be an ample line bundle on X, and $x \in X(\bar{k})$. By slightly modifying the usual definition of approximation exponent on $\mathbf{P}^1$, we define a new invariant $\alpha_x(L) \in (0, \infty]$ which measures how quickly rational points accumulate around x, as measured by L.

The central theme of the talk is the interrelations between $\alpha_x(L)$ and the Seshadri constant $\epsilon_x(L)$ which measures the local positivity of L near x. In particular, the classic approximation theorem of Klaus Roth on $\mathbf{P}^1$ generalizes as an inequality between α_x and ϵ_x valid for all projective varieties. This is joint work with David McKinnon (Waterloo).