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**Recent Research in Econometrics**  
**Recherches récentes en économétrie**  
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**XIN LIANG**, McGill University

*Necessary and sufficient conditions for identification and estimability of linear parameters*

We study the relationship between estimability and identifiability of linear parameters in partially linear models. A model is partially linear for the parameter vector  $\beta$  if the conditional distribution of the data given  $X$  depends on  $\beta$  through  $X\beta$ , where  $X$  is a known matrix. We focus here on situations where  $X$  may not have a full-column rank, and  $X\beta$  can be interpreted as an identifiable parameter. Besides linear regressions, partially linear models include several widely used statistical models: generalized linear models and linear mixed models, median regression, quantile regressions, various discrete choice models (such as probit and Tobit models), single index models, etc. We observe that usual conditions for parameter estimability in linear regressions – *a fortiori* in partially linear models – are not necessary for identification, so estimability is not equivalent to identifiability. In the context of a general likelihood model (which may not be partially linear), we give a necessary and sufficient condition for identification of a transformation of model parameters. The proposed partial identification condition involves a general form of (potentially nonlinear) separability. This result is then applied to characterize the identification of an arbitrary vector  $Q\beta$  in a partially linear model. Several equivalent partial identifiability conditions are provided, and close-form representations are provided for the corresponding “identification sets” as linear subspaces of the parameter space. The proposed identifiability conditions include a number of easily interpretable conditions not previously supplied in the literature on estimability in linear regression.

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**MIRZA TROKIC**, McGill University

*Regulated Variance Ratio Unit Root Tests*

Regulated (bounded) integrated time series are of significant practical importance. Although regulated integrated series are characterized by asymptotic distributions which differ substantially from their unregulated counterparts, inferential exercises continue to be performed with complete disregard for this feature of time series data. This article aims to bridge this gap by proposing the variance ratio statistic of Nielsen (2009) in the case of regulated series. The article develops asymptotic distribution for the standard and OLS detrended versions of the statistic. In the unbounded case this statistic offers a means of improving statistical power of the test by choosing the fractional integration parameter  $d$  to be as small as possible. What this paper demonstrates is that no such template exists when the series is bounded. Choices of  $d$  in the regulated case depends heavily on the length, direction, and nature of the bounding interval. In cases where the bounding interval is sufficiently wide so that the problem may be considered “unbounded”, the results in Nielsen (2009) are replicated. In all other cases, the regulated variance ratio statistic suffers from very low power which in most cases of interest decreases to zero as one moves away from the unit root null hypothesis into the stationary alternative hypothesis. Finally, this paper extends the results of Cavaliere and Xu (2011) by introducing what seems to be the first theoretical justification for the asymptotic distribution of regulated integrated time series with a linear trend.

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**PUREVDORJ TUVAANDORJ**, McGill University

*Maximum Likelihood Estimation and Inference in Possibly Unidentified Models*

The validity of standard distributional approximation in regular statistical model critically hinges upon the identifiability of the model. Lack of identification imposes strong limitations on the construction of estimators and test statistics for nonidentifiable parameters with desirable properties. Motivated by the observation that identification failure does not preclude the possibility of making valid inference on the identifiable part of the model, the present paper studies identification, estimation and hypotheses testing in possibly unidentified parametric models. We give necessary and sufficient conditions for local identifiability of a parametric function in terms of its Jacobian matrix with respect to the parameter of the model and the Fisher information

matrix. Based on local asymptotic analysis, it is shown that despite the identification failure the score and likelihood ratio statistics for testing hypothesis on the identifiable parameter have chi-square limiting distribution with degrees of freedom equal to the number of restrictions under certain regularity conditions. Moreover, stochastic dominance relations between various test statistics are provided.