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Necessary and sufficient conditions for identification and estimability of linear parameters

We study the relationship between estimability and identifiability of linear parameters in partially linear models. A model is partially linear for the parameter vector β if the conditional distribution of the data given X depends on β through $X\beta$, where X is a known matrix. We focus here on situations where X may not have a full-column rank, and $X\beta$ can be interpreted as an identifiable parameter. Besides linear regressions, partially linear models include several widely used statistical models: generalized linear models and linear mixed models, median regression, quantile regressions, various discrete choice models (such as probit and Tobit models), single index models, etc. We observe that usual conditions for parameter estimability in linear regressions – *a fortiori* in partially linear models – are not necessary for identification, so estimability is not equivalent to identifiability. In the context of a general likelihood model (which may not be partially linear), we give a necessary and sufficient condition for identification of a transformation of model parameters. The proposed partial identification condition involves a general form of (potentially nonlinear) separability. This result is then applied to characterize the identification of an arbitrary vector $Q\beta$ in a partially linear model. Several equivalent partial identifiability conditions are provided, and close-form representations are provided for the corresponding "identification sets" as linear subspaces of the parameter space. The proposed identifiability conditions include a number of easily interpretable conditions not previously supplied in the literature on estimability in linear regression.