
Contributed Papers
Communications libres
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FABRICE COLIN, Université Laurentienne / Laurentian University
Generalized Fountain Theorem and Application to the Semilinear Schrödinger Equation

Fountain theorems and their variants have proven to be effective tools in studying the existence of infinitely many solutions of partial differential equations. By using the degree theory and the τ -topology of Kryszewski and Szulkin, we establish a version of the Fountain Theorem for strongly indefinite functionals. This abstract result will be applied for studying the existence of infinitely many solutions of two strongly indefinite semilinear problems including the semilinear Schrödinger equation.

IBRAHIMA DIONE, Université Laval
Penalty/finite element approximations of slip boundary conditions and Babuska's paradox

The penalty method is a classical and widespread method for the numerical treatment of constrained problems such as unilateral contact problems and problems with Dirichlet boundary conditions. It provides an alternative approach to constrained optimization problems which avoids the necessity of introducing additional unknowns in the form of Lagrange multipliers. In the case of slip boundary conditions for fluid flows or elastic deformations, one of the main obstacle to their efficiency and to their mathematical analysis is that a Babuska's type paradox occurs.

Observed first by Sapondzyan [2] and Babuska [1] on the plate equation in a disk with simple support boundary conditions, Babuska's paradox can be stated as follows: on a sequence of polygonal domains converging to the domain with a smooth boundary, the solutions of the corresponding problems do not converge to the solution of the problem on the limit domain.

Our presentation will focus on the finite element approximation of Stokes equations with slip boundary conditions imposed with the penalty method in two and three space dimensions. For a polygonal or polyhedral boundary, we prove convergence estimates in terms of both the penalty and discretization parameters. In the case of a smooth curved boundary, we show through a numerical example that convergence may not hold due to a Babuska-type paradox. Finally, we propose and test numerically several remedies.

[1] Babuska I. and Pitkaranta J. SIAM J. MATH. ANAL., 21 (1990) 551-576.

[2] Sapondzyan O.M. Akad. Nauk Armyan. SSR. Izv. Fiz.-Mat. Estest. Tehn. Nauki, 5:29-46, 1952.

SAFOUHI HASSAN, University of Alberta
New Formulae for Differentiation and Techniques in Numerical Integration

We present new formulae, called the Slevinsky-Safouhi's formulae (SSF) I and II [1] for the analytical development of derivatives. The SSF, which are analytic and exact, represent the derivative as a discrete finite sum involving coefficients that can be computed recursively and they are not subject to any computational instability.

There are numerous applications in science and engineering for special functions and higher order derivatives. As an example, the nonlinear G transformation has proven to be a very powerful tool in numerical integration [3]. However, this transformation requires higher order derivatives of the integrands for the calculation, which can be a severe computational impediment.

As examples of applications of the SSF, we present higher order derivatives of Bessel functions which are prevalent in oscillatory integrals and provide tables illustrating our results. We also present an efficient recursive algorithm for the implementation of the G transformation. The incomplete Bessel function is presented as an example of application. Lastly, we present a generalized and formalized integration by parts to create equivalent representations to some challenging integrals. As an example of application, we present the Twisted tail.

- [1] R. M. Slevinsky and H. Safouhi. New formulae for higher order derivatives and applications. *J. Comput. App. Math.*, 233:405–419, 2009.
- [2] H. L. Gray and S. Wang. A new method for approximating improper integrals. *SIAM J. Numer. Anal.*, 29:271–283, 1992.
- [3] R. M. Slevinsky and H. Safouhi. The S and G transformations for computing three-center nuclear attraction integrals. *Int. J. Quantum Chem.*, 109:1741–1747, 2009.

PATRICK LACASSE, Université Laval

Résolution d'un problème d'élasticité avec contact sans frottement par stratégie de contraintes actives et algorithme itératif.

On s'intéresse au problème de calculer la déformation qu'un corp élastique entrant en contact avec un corp rigide. Il s'agit donc d'optimiser une fonctionnelle non linéaire sous une contrainte inégalité également non linéaire. Dans un premier temps, ces contraintes sont reportés sur un espace de multiplicateurs de Lagrange. Une stratégie dite de contraintes actives permet par la suite de transformer le problème en une suite de problèmes avec contrainte égalité. Selon la méthode classique de Newton, ce système est linéarisé, conduisant à un système matriciel par bloc. Ce dernier est finalement résolu par une approche itérative tirant profit de la factorisation de ce système particulier.

SOPHIE LÉGER, Université Laval

An updated Lagrangian method for very large deformation problems

The use of the finite element method is quite widespread for the analysis of large deformation problems, notably for the calculation of tire deformation. In this case and in many others, a good numerical method is essential. Industrial partners expect accurate, efficient and robust methods, and all of this preferably at a low computational cost.

When using a Lagrangian point of view in the finite element method for the resolution of large deformation problems, the mesh elements can become severely distorted over time. This can lead to numerical instabilities and slow convergence. To avoid this problem, frequent remeshing of the domain during the computation becomes necessary in order to optimize the quality of the mesh and thus improve convergence. In an updated Lagrangian framework, the deformation gradient tensor, which is key for the calculation, has to be transferred from the old mesh to the new mesh after each remeshing step. In this presentation, we will compare different transfer techniques and show which one seems to be more efficient and give the best results.

Numerical continuation methods have proved to be very powerful tools when dealing with very nonlinear problems. When combining both a good remeshing algorithm and a good transfer method for the deformation gradient tensor with the Moore-Penrose continuation method, we will show that very large levels of deformation can be attained and that the combination of all these tools leads to a very stable and efficient updated Lagrangian algorithm.

TRUEMAN MACHENRY, York University

Permanents, Determinants, Integer Sequences and Isobaric Polynomials

ABSTRACT. In this paper we construct two types of Hessenberg matrices with the properties that every weighted isobaric polynomial (WIP) appears as a determinant of one of them, and as the permanent of the other. Every integer sequence which is linearly recurrent is representable by (an evaluation of) some linearly recurrent sequence of WIPs. WIPs are symmetric polynomials written on the elementary symmetric polynomial basis. Among them are the generalized Fibonacci polynomials and the generalized Lucas polynomials, which already have these sweeping representing properties. Among the integer sequences discussed are the Chebychev polynomials of the 2nd kind, the Stirling numbers of the 1st and 2nd kind, the Catalan numbers, and the triangular numbers, as well as all sequences which are either multiplicative arithmetic functions or additive arithmetic functions.

ODILE MARCOTTE, CRM- UQAM

On the maximum orders of an induced forest, an induced tree, and a stable set

Let G be a connected graph, n the order of G , and f (resp. t) the maximum order of an induced forest (resp. tree) in G . We give upper bounds for $f-t$ (depending upon the value of n) and show that these bounds are tight. We give similar results for the difference between the stability number of G and the maximum order of an induced tree in G .

STEPHANIE PORTET, University of Manitoba

Dynamics of length distributions of in vitro intermediate filaments

Intermediate filaments are one of the cytoskeleton components. The cytoskeleton is an intracellular structure made of proteins polymerized in filaments that are organized into networks in the cytoplasm. Here a general method is given to study the dynamics of length distributions of filaments described as linear macromolecules. An aggregation model with explicit expression of association rate constants depending on the properties of interacting objects is considered. A set of hypotheses on the geometry and properties of interacting macromolecules is considered, leading to a collection of models. Fitting of model responses to experimental data yields the best-fit for each model in the collection. By using model selection, the more appropriate model to represent the assembly at a given time point is identified. Hence, conclusions on the object properties can be drawn.

ANTHONY SHAHEEN, California State University, Los Angeles

A Brief Introduction to Expanders and Ramanujan Graphs

Think of a graph as a communications network. Putting in edges (e.g., fiber optic cables, telephone lines) is expensive, so we wish to limit the number of edges in the graph. At the same time, we would like the communications network to be as fast and reliable as possible. We will see that the quality of the network is closely related to the eigenvalues of the graph's adjacency matrix. Essentially, the smaller the eigenvalues are, the better the communications network is. It turns out that there is a bound, due to Alon, Serre, and others, on how small the eigenvalues can be. This gives us a rough sense of what it means for graphs to represent "optimal" communications networks; we call these Ramanujan graphs. Families of k -regular Ramanujan graphs have been constructed in this manner by Lubotzky, Sarnak, and others whenever $k-1$ equals a power of a prime number. No one knows whether families of k -regular Ramanujan graphs exist for all k .

CHUNHUA SHAN, York University

Finite cyclicity of hh-graphics with a triple nilpotent singularity of codimension 4

In 1994, Dumortier, Roussarie and Rousseau launched a program aiming at proving the finiteness part of Hilbert's 16th problem for the quadratic system. For the program, 121 graphics need to be proved to have finite cyclicity. In this presentation, I will report our effort to show that some hh-graphics through a triple nilpotent singularity of codimension 4 have finite cyclicity. This is an in progress joint work with professor Christiane Rousseau and professor Huaiping Zhu.

IICKHO SONG, Korea Advanced Institute of Science and Technology

An Extension of the Vandermonde Convolution Formula

As an extension of the Vandermonde Convolution $\sum_{m=0}^{\gamma} \binom{\alpha}{\gamma-m} \binom{\beta}{m} = \binom{\alpha+\beta}{\gamma}$, an explicit expression for the sum $\sum_{m=0}^{\gamma} m(m-1)\cdots(m-\zeta+1) \binom{\alpha}{\gamma-m} \binom{\beta}{m}$ is obtained, where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ denotes the binomial coefficient. Some examples for the application of the result are considered.