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Inclusive prime number races

Let $\pi(x; q, a)$ denote the number of primes up to x that are congruent to $a \pmod{q}$. A "prime number race", for fixed modulus q and residue classes a_1, \dots, a_r , investigates the system of inequalities $\pi(x; q, a_1) > \pi(x; q, a_2) > \dots > \pi(x; q, a_r)$. We expect that this system should have arbitrarily large solutions x , and moreover we expect the same to be true no matter how we permute the residue classes a_j ; if this is the case, the prime number race is called "inclusive". As it happens, the explicit formula for $\pi(x; q, a_j)$ allows us to convert prime number races into problems about sums of infinitely many random variables and the analogous inequalities among them.

Rubinstein and Sarnak proved conditionally that every prime number race is inclusive; they assumed not only the generalized Riemann hypothesis but also a strong statement about the linear independence of the zeros of Dirichlet L -functions. On the other hand, Ford and Konyagin showed that prime number races could fail to be inclusive if the generalized Riemann hypothesis is false. We will discuss these results, as well as some work in progress with Nathan Ng where we substantially weaken the second hypothesis used by Rubinstein and Sarnak.