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Perturbations to non-classical shocks in non-convex systems of conservation laws

In convex systems of nonlinear conservation laws, the class of weak solutions that satisfy an entropy condition is equivalent to the class of solutions that can be obtained as limits of diffusive regularizations. This equivalence breaks down for non-convex conservation laws, like the equations of ideal magnetohydrodynamics or of certain thin film flows. In fact, the work of Bianchini and Bressan has shown that diffusive regularizations are sufficiently discriminating to allow the identification of a class of well-posed solutions even in the large family of first order hyperbolic Cauchy problems. Unfortunately, non-convex systems often appear as models with neglected high-order diffusive and dispersive physics and the previous result is insufficient.

For non-convex systems, imposing the sign AND the rate of entropy production is essentially equivalent to choosing solutions that are limits to specific diffusive-dispersive regularizations. This correspondence has been demonstrated at the level of Riemann problems but has yet to be established for non-convex conservation laws with small total variation initial data.

In this joint work with LeFloch, we exploit his theory of kinetic functions in order to establish the existence of solutions to small perturbations of non-classical shocks in systems of non-convex conservation laws, i.e. shocks that are not limits of diffusive regularizations. Given that stability and uniqueness have already been established for small total variation initial data, this work is one step towards a well-posedness theory for non-convex conservation laws, and ultimately, for hyperbolic Cauchy problems subject to general regularizations.