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Distribution of randomly propagated Schrödinger eigenfunctions

This is joint work with Dmitry Jakobson and John Toth. Let (M, g_0) be a compact Riemmanian manifold, and $V \in C^\infty(M)$. Let $P_0(h) := -h^2\Delta_{g_0} + V$, be the semiclassical Schrödinger operator for $h \in (0, h_0]$. If φ_h is an L^2-normalized eigenfunction of $P_0(h)$, then $\int_A |\varphi_h(x)|^2 dv_{g_0}(x)$ is interpreted as the probability that a quantum particle of energy $\sim 1/h^2$ belongs to $A \subset M$. For a quantum particle with initial state φ_h, its evolution at time t is described by the same probabilty density since $|e^{-\frac{it}{h}P_0(h)}\varphi_h| = |\varphi_h|$. However, since real life systems are usually affected by "noise", the time evolution is better decribed by the state

$$\varphi_h^{(u)}(x) = e^{-\frac{it}{h}P_u(h)}\varphi_h$$

where $P_u(h)$ is some small perturbation of $P_0(h)$.

In this talk we consider a smooth family of perturbations g_u of the reference metric g_0 for $u \in \mathcal{B}^k(\varepsilon) \subset \mathbb{R}^k$ of radius $\varepsilon > 0$, and consider the perturbed Schrödinger operators $P_u(h) := -h^2\Delta_{g_u} + V$. For $t > 0$ small, we study the moments of the real part of the perturbed eigenfunctions regarded as random variables

$$Re\left(\varphi_h^{(\cdot)}(x)\right) : \mathcal{B}^k(\varepsilon) \to \mathbb{R} \qquad \text{for } x \in M.$$