
Algebraic Number Theory
Théorie algébrique des nombres
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MICHAEL BENNETT, University of British Columbia

Effective solution of norm form equations

Thanks to the Subspace Theorem of Wolfgang Schmidt, we have a solid understanding of norm form equations over number fields, at least from a “qualitative” viewpoint. The situation is much less satisfactory if we desire to solve such equations “effectively”, however, in all but the simplest cases. In this talk, I will sketch recent work on applications of linear forms in logarithms to such problems, generalizing work of Vojta, and applications of these results.

MILJAN BRAKOCEVIC, McGill University

Two variable anticyclotomic p -adic L -functions for p -adic families of modular forms

For the anticyclotomic p -adic Rankin–Selberg L -function attached to a

Hecke eigenform and an imaginary quadratic

field we explain the construction of the two-variable p -adic L -function by introducing the second p -adic variable by considering Hida and Coleman families of Hecke eigenforms parametrized by the weight.

FRANCESC CASTELLA, McGill University

On the p -adic variation of Heegner points

We will report on our recent work on the relation between Heegner points and p -adic L -functions, both made to vary in p -adic families. By Kolyvagin’s “method of Euler systems”, this leads to a number of applications, in particular to the arithmetic of modular elliptic curves.

HUGO CHAPDELAINE, U. Laval

Archimedean Stark conjectures and real analytic Eisenstein series

In this talk we will present a construction of real analytic Eisenstein series $E(z, s)$ attached to a totally real field K , $E(z, s)$ being real analytic in z and holomorphic in s . We will present a precise formula for its Fourier series expansion around z . Having such an explicit formula at our disposal, we will then prove a functional equation which relates $E(z, s)$ to its so-called “dual Eisenstein series” $E^*(z, 1 - s)$. It turns out that the constant term of this Fourier series is a partial zeta function $\zeta(s)$ in the complex variable s weighted by a sign character. In the special case when $\text{ord}_{s=0}(\zeta(s)) = 1$, it is expected that $\zeta'(0)$ is equal to the logarithm of a global unit in an abelian extension of K . In order to get some insights about a possible solution of this outstanding conjecture, we will present a (classical) proof of this conjecture in the special case when $K = \mathbb{Q}$ which involves Cauchy’s classical residues theorem.

IMIN CHEN, Simon Fraser University

Ramification of Drinfeld modules

I will survey a number of results regarding bounding the ramification of the torsion fields of Drinfeld modules, with applications to Serre large image type results for the associated Galois representations.

CLIFTON CUNNINGHAM, University of Calgary

A geometric and categorical description of complete Langlands parameters for quasi-split p -adic groups

Let G be a connected reductive quasi-split algebraic group over a p -adic field. In this talk we introduce an abelian category of equivariant perverse sheaves on an ind-variety built from ${}^L G$, the L-group for G , and show that there is a canonical bijection between isomorphism classes of simple objects in this category and complete Langlands parameters. Joint work with Pramod Achar, Masoud Kamgarpour and Hadi Salmasian. This group is currently working on a proof that this category is Koszul.

This geometric and categorical approach to complete Langlands parameters suggests a geometric and categorical approach to irreducible admissible representations and the local Langlands Correspondence itself, which is already realised in joint work with David Roe when $G = GL(1)$, and under construction for other algebraic tori. Time permitting, I will also say a few words about this work.

JOCHEN GARTNER, McGill University

Arithmetic curves of type $K(\pi, 1)$ and higher Massey products

If k is a number field, p an odd prime number and S a finite set of primes of k containing the set S_p of primes above p , the arithmetic curve $\text{Spec}(\mathcal{O}_k \setminus S)$ is a $K(\pi, 1)$ for p , i.e. the pro- p -completion of its étale homotopy type is weakly equivalent to the Eilenberg-MacLane space of $\pi_1^{et}(\text{Spec}(\mathcal{O}_k \setminus S)(p))$, the Galois group of the maximal pro- p -extension of k unramified outside S . We discuss arithmetic consequences of the $K(\pi, 1)$ -property in the more difficult tame case (i.e. $S \cap S_p = \emptyset$) due to A. Schmidt and show how the first explicit examples have been obtained by J. Labute using the theory of mild pro- p -groups. We investigate how these groups can be constructed using higher cohomological Massey products and give an arithmetic interpretation for $k = \mathbb{Q}$ in terms of certain analogues of p -th power symbols.

MATTHEW GREENBERG, University of Calgary

p -arithmetic and completed cohomology of quaternionic Shimura varieties

In this report on joint work with Samit Dasgupta, I will describe a relationship between classes in the cohomology of p -arithmetic groups and intertwining operators between smooth or p -adic representations of $GL_2(F_p)$, F a totally real field, and classical or completed cohomology groups of p -towers of quaternionic Shimura varieties over F . This relationship can be used generalize some results of Breuil and Emerton concerning L -invariants and p -adic L -functions to the setting of Hilbert modular forms.

ERNST KANI, Queen's University

Tensor Products of Galois Representations

Let $\rho_{A_i, \ell} : G_K \rightarrow \text{Aut}(V_\ell(A_i))$ be the ℓ -adic Galois representation attached to an abelian variety A_i/K , and let $\tau_{A_i, \ell} : \text{End}(A_i) \otimes \mathbb{Q}_\ell \xrightarrow{\sim} \text{End}_{\mathbb{Q}_\ell[G_K]}(V_\ell(A_i))$ be the canonical isomorphism (Tate/Faltings). The purpose of this talk is to study properties of the tensor product $\rho_{A_1, \ell} \otimes \rho_{A_2, \ell}$ of two such representations, particularly in view of the following question: when is $\tau_{A_1, \ell} \otimes \tau_{A_2, \ell} : \text{End}(A_1) \otimes \text{End}(A_2) \otimes \mathbb{Q}_\ell \rightarrow \text{End}_{\mathbb{Q}_\ell[G_K]}(V_\ell(A_1) \otimes V_\ell(A_2))$ an isomorphism? (This question is related to Tate's Conjecture for codimension 2 cycles on products of abelian varieties.) In this talk I will give a solution in the case when $K = \mathbb{Q}$ and $A_i = A_{f_i}$ is a modular abelian variety attached to a weight 2 newform f_i on $\Gamma_1(N_i)$. If time permits, I will also discuss mod ℓ analogues.

PAYMAN KASSAEI, Kings College London

Modularity Lifting via Analytic continuation of Hilbert modular forms

In his foundational work on the theory of p -adic modular forms, N. Katz observed that there is a positive lower bound for the "growth condition" of an overconvergent p -adic modular eigenform with nonzero U_p -eigenvalue. In more modern language, this states that any such form can be analytically continued from its initial domain of definition to a not "too small" region of the rigid analytic modular curve. Years later, K. Buzzard, by adding $\Gamma_0(p)$ to the level, proved that such forms can be further extended to a certain "large" region of the modular curve. These results were used by Buzzard and Taylor to prove modularity lifting results which led to a proof of certain cases of the Strong Artin conjecture.

It has been known for a while how to extend these results to the Hilbert case when p is split in the totally real field of degree $g > 1$, as the problem looks formally like a product of g copies of the modular curve case. In the inert case, however, a mixing happens that fundamentally changes the nature of the problem. In this talk, I will explain new results on domains of automatic analytic continuation for overconvergent Hilbert modular forms in the case p is unramified in the totally real field. These results can be used to prove many cases of the strong Artin conjecture for Hilbert modular forms. Some of the work that will be presented is joint with Sasaki and Tian.

MANFRED KOLSTER, McMaster University

The Coates-Sinnott Conjecture

The Coates-Sinnott Conjecture was formulated in 1974 as a K-theory analogue of Stickelberger's Theorem and proven for K2 for abelian number fi

elds up to 2-torsion. In this talk we present recent results about the general situation of higher K-groups, arbitrary relative abelian extensions of number fi

elds and all primes including 2. The most complete general results for all primes are due to R. Taleb, and in some more specific situations to Taleb and myself.

STEPHEN KUDLA, University of Toronto

An alternative description of Drinfeld space and applications

Let F be a finite extension of \mathbb{Q}_p of degree d and let E/F be a quadratic extension with ring of integers O_E . In this lecture I will explain how Drinfeld's formal 'upper half plane' also arises as a moduli space for p -divisible groups of dimension $2d$ and height $4d$ with an action of O_E and a polarization which may be principal or not depending on whether E/F is ramified or not. If time permits, I will explain how to use this model of Drinfeld space to give new examples of p -adic uniformization of certain Shimura varieties. This is joint work with Michael Rapoport.

JOHN LABUTE, McGill University

Linking Numbers, Mild Groups and the Fontaine-Mazur Conjecture

Let p be a prime, let S be a finite set of primes $q \equiv 1 \pmod{p}$ but $q \not\equiv 1 \pmod{p^2}$ and let G_S be the Galois group of the maximal p -extension of \mathbb{Q} unramified outside of S . If ρ is a continuous homomorphism of G_S into $GL_2(\mathbb{Z}_p)$ we use the Koch presentation of G_S and the theory of mild pro- p -groups to show that if $p > 3$ then, under certain conditions on the linking numbers of the primes in S , either $\rho = 1$ or $\rho(G_S)$ is a Sylow p -subgroup of $SL_2(\mathbb{Z}_p)$. Under certain conditions on S with $|S| = 2, 3$, we show that $\rho = 1$.

ANTONIO LEI, McGill University

Beilinson-Flach elements for the Rankin-Selberg convolution

I will talk about my joint work with David Loeffler and Sarah Zerbes on the construction of cohomology classes for the Rankin-Selberg convolution of two weight-two modular forms, that is, to construct a collection of elements in $H^1(\mathbb{Q}(\mu_m), V_f \otimes V_g)$ where f and g are modular forms of weight two and V_f and V_g are the corresponding Deligne representations. I will also talk about how such construction is related to Perrin-Riou's conjecture on the existence of an Euler system for $V_f \otimes V_g$ and other applications.

FRANTISEK MARKO, Pennsylvania State University

Congruences of Ankeny-Artin-Chowla type and p -adic class number formula revisited

The purpose of this talk is to interpret results of Jakubec, Jakubec-Lassak, Marko and Jakubec-Marko on congruences of

Ankeny-Artin-Chowla type for cyclic totally real fields as an elementary version of the p -adic class number formula modulo powers of p . Explicit formulas for quadratics and cubic fields will be given.

JAN MINAC, Western University

Abel-Galois T.V. Channel D8 Interview of V.I.S.G.G.

A possible characterization of absolute Galois groups among profinite groups still seems to be a very difficult task.

Recently however several remarkable developments have opened up new ways of exploration. These include the Rost-Voevodsky proof of the Bloch-Kato conjecture, the advances of F. Bogomolov, F. Pop and Y. Tschinkel, on a birational abelian program, and some advances on the structure and surprising anabelian character of rather small quotients of finite exponents of absolute Galois groups. The interview and its aftermath will focus on the latter explorations carried out with S. Chebolu, I. Efrat, J. Swallow and A. Topaz.

CHUNG PANG MOK, McMaster University

Endoscopic classification of representations for quasi-split unitary groups

We report on the work on endoscopic classification for quasi-split unitary groups, following Arthur's methods. We will highlight some local and global results that are corollaries of the theory.

KUMAR MURTY, University of Toronto

Growth of the Selmer group of an Abelian variety with CM

We consider an Abelian variety A of complex multiplication defined over a number field F , and study the growth of the p -rank of the Selmer group of A in certain classes of infinite p -extensions of F . This is joint work with Meng Fai Lim.

SUJATHA RAMDORAI, UBC

Congruences between p -adic L -values

Noncommutative Iwasawa theory predicts congruences between twisted p -adic L -values arising from Artin representations. We shall discuss the background and present numerical evidence of such congruences.

GLENN STEVENS, Boston University

The Hodge-Tate sequence and overconvergent p -adic modular sheaves

Using Faltings' theory of the Hodge-Tate sequence of an abelian scheme we give a functorial construction of "modular sheaves" Ω^κ , where κ is a not-necessarily integral weight, attached to abelian schemes on which the canonical subgroup exists. These sheaves generalize the integral powers, ω^k , of the sheaf ω of relative differentials on a modular curve. Global sections of Ω^κ provide geometric realizations of overconvergent automorphic forms of non-integral weight. Applications of this approach to the theory of p -adic Hilbert modular forms will be described. This is joint work with Fabrizio Andreatti and Adrian Iovita.

MAK TRIFKOVIC, University of Victoria

The Bost-Connes Functor

Alain Connes has proposed solving Hilbert's 12th problem by constructing points on certain 'non-commutative varieties'. Such a variety is given by a C^* -dynamical system, the Bost-Connes system, made out of the adèles of a number field. The talk will discuss the construction, along with recent work by the presenter, M. Laca and S. Neshveyev which turns this construction into a functor from algebraic number fields to C^* -dynamical systems.

JOHN VOIGHT, University of Vermont

Kronecker's Jugendtraum and power series expansions of modular forms

In Kronecker's Jugendtraum, abelian extensions of an imaginary quadratic field are constructed by special values of modular functions on classical modular curves. In practical terms, this allows one to find generating polynomials for number fields arising from class field theory using numerical methods. We extend Kronecker's Jugendtraum to CM number fields in this explicit way by computing special values of functions on Shimura curves over totally real fields. To do so, we exhibit a method to numerically compute power series expansions of modular forms on a cocompact Fuchsian group, using the explicit computation a fundamental domain and linear algebra.

SOROOSH YAZDANI, University of Lethbridge

Local Szpiro conjecture and rational points on $X_E(6k)$

Motivated by the Szpiro conjecture, we conjecture that for any prime p , any integer M , and any integer $l > 6$, if E is a semistable elliptic curve with minimal discriminant $p^r M^l$, then $r \leq 6$. We prove this conjecture for many primes p and $l = 6k$ by looking for perfect powers in certain elliptic nets.