Applied Mathematics
Mathématiques appliquées
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ALEXANDER BIHLO, Centre de recherches mathématiques, Université de Montréal
Invariant subgrid-scale closure schemes for turbulence modeling

The understanding and modeling of turbulence is one of the remaining great challenges in classical physics. In numerical simulations of turbulent flows one generally faces the problem of not being able to resolve all the scales that are relevant for the accurate prediction of the flow. Thus, a subgrid-scale closure scheme has to be introduced that mimics the effects of the unresolved scales. The use of an artificial subgrid-scale closure model bears the risk of destroying the geometric properties of the differential equations describing the fluid, which ultimately can lead to spurious effects in the computed numerical solutions.

In this talk we will focus on the systematic construction of turbulence models that preserve the Lie symmetries of the incompressible Euler or Navier-Stokes equations. The method we introduce relies on equivariant moving frames, which can be used to send an existing non-invariant subgrid-scale model to an invariant closure scheme. Numerical examples will be given in order to demonstrate that invariant turbulence models can yield realistic results in turbulence simulations.

LYDIA BOUROUIBA, Massachusetts Institute of Technology
Nonlocal energy transfers in rotating homogeneous turbulence

Turbulent flows subject to solid-body rotation are known to generate large scale two-dimensional columnar vortices. The dominant mechanisms leading to the accumulation of energy in the two-dimensional columnar vortices remain undetermined. Here, I will discuss scale-locality of the nonlinear interactions directly contributing to the growth of the two-dimensional columnar structures observed in the intermediate Rossby number regime. Implications for existing theories of rotating flows will be discussed.

ELENA BRAVERMAN, University of Calgary
On the competition of spatially distributed populations choosing different diffusion strategies

For spatially distributed models of population dynamics described by reaction-diffusion equations with unequally distributed resources, we consider various types of diffusion and compare models with regular diffusion and the strategy which favors the movement in the direction of the highest per capita available resources. For several populations which only differ by the diffusion coefficient, there are theoretical results that the slowest population has an evolutionary advantage. We demonstrate that under quite non-restrictive conditions, the population with the directed diffusion strategy survives in the competition with a similar population which adopted the regular diffusion, while the latter population extincts, independently on the diffusion coefficient. This is a joint work with my PhD students L. Korobenko and Md. Kamrujjaman.

BERNARD BROOKS, Rochester Institute of Technology
A two-population competition model for a finite natural resource

A system of coupled differential equations is used to model the interaction between humans and their natural island environment. The two human populations differ only in their harvesting rates. They compete for a finite natural resource whose growth rate is logistic. Each population can coexist stably with the natural environment in the total absence of the other type of human. Thus there exist two equilibria of the coupled three-differential-equation system; one with only the high-rate harvesters and one with only the low-rate harvesters. It will be shown though the nested box technique that the equilibrium with only high-rate harvesters is stably resistant to invasion from low-rate harvesters whereas the equilibrium with only low-rate harvesters is unstable and susceptible to an invasion of high-rate harvesters. In the end greed wins.

XIAOWEN CHANG, McGill University
Why is the LLL reduction useful for solving an integer least squares problem

In some applications such as GPS and communications, there is a linear model, which involves an unknown integer parameter vector. The common method to estimate the integer parameter vector is to solve an integer least squares (ILS) problem, also referred to as a closest vector problem. A typical approach to solving an ILS problem is the so called sphere decoding, a discrete search method. To make a sphere decoder faster, the well-known Lenstra, Lenstra and Lovasz (LLL) reduction is often used as preprocessing. As a general ILS problem is NP-hard, for some applications, an approximate solution, which can be produced quickly, is computed instead. One often used approximate solution is the Babai point, the first integer point found by a typical sphere decoder. In order to verify whether an estimator is good enough for a practical use, one needs to find the probability of the estimator being equal to the true integer parameter vector, which is referred to as success probability. In addition to making the search process faster, it has been observed that the LLL reduction can also improve the success probability of the Babai point. But there has been no rigorous theory about either observation so far. In this talk we will show rigorously in theory why both are true.

This is joint work with Jinming Wen and Xiaohu Xie

LOUIS-PHILIPPE SAUMIER DEMERS, University of Victoria
Optimal Transport for Particle Image Velocimetry

We present a new method for particle image velocimetry, a technique using successive laser images of particles immersed in a fluid to measure the velocity field of the fluid flow. The main idea is to recover this velocity field via the solution of the L^2-optimal transport problem associated with each pair of successive distributions of tracers. We model the tracers by a network of Gaussian-like distributions and derive rigorous bounds on the approximation error in terms of the model's parameters, i.e. the uncertainty in the position of the particles and the noise level in the measurements. We also display the results of numerical experiments based on synthetic flow fields. The numerical solution is obtained by employing Newton's method to solve the Monge-Ampère equation associated with the transport problem.

HAROLD HASTINGS, Hofstra University
Can averaging stabilize complex systems?

Averaging (called diversification in some contexts) has the potential to reduce variability. For example, the variance of the average of n independent, identically distributed (i.i.d.) random variables, of variance s2, is s2/n. Averaging can work to reduce the variance below a given threshold of acceptability provided that the variance (and in some cases also higher moments) is small or the system size is large; otherwise, in the case of fat-tailed distributions or effectively small systems, averaging will fail. Here we explore the meaning of this statement in the context in a variety of complex systems.

RONALD HAYNES, Memorial University of Newfoundland
A Parallel Space-Time Approach for the Numerical Solution of Partial Differential Equations

There is a continuing need to pursue the development, implementation and theoretical analysis of algorithms for the numerical solution of time dependent PDEs particularly suited to today's increasingly complex problems of interest. Moreover, there is an opportunity to study algorithms designed to take advantage of evolving computing hardware - available commodity clusters, hybrid CPU-GPU systems and even desktop machines with 4-24 cores. Such algorithms consist of three modules: (1) a procedure to step forward in time, (2) the computation of a new spatial mesh as required, and (3) the solution of the (physical) PDE on the newly constructed mesh. Moving mesh methods, developed over the last 30 years, have proven to be a robust and efficient choice to track solutions of PDEs which evolve over disparate space and time scales. Domain decomposition (DD) parallelizes a computation by partitioning the spatial domain into subdomains. The solution on each subdomain is computed by individual processors or cores. With appropriate conditions to transmit solution information between cores, the subdomain solutions can be rapidly combined to give a solution to the original problem. The application of DD methods for the physical

PDE, step (3) above, is well established. Here we will consider the application of DD to the PDE based mesh generation problem used in the moving mesh method. And finally we present some recent work on the Revisionist Integral Deferred Correction approach which is a relatively easy way to add small scale parallelism (in time) to the solution of time dependent PDEs.

DMITRY KOLOMENSKIY, Department of Mathematics and Statistics, McGill University / CRM
Similarity solutions for unsteady flows near front and rear stagnation points

Unsteady flows near stagnation points on a cylindrical body immersed in a viscous incompressible fluid are considered. This problem admits similarity solutions, which are exact solutions of the Navier-Stokes equations, having a boundary-layer character similar to that of classical steady forward stagnation point flow. The velocity profiles are obtained by numerical integration of a non-linear ordinary differential equation. A wide range of possible behaviour is revealed, depending on the flow direction and acceleration. For the forward-flow situation, the solution is unique for the accelerating case, but bifurcates for modest deceleration, while for sufficient rapid deceleration there exists a one-parameter family of solutions. For the rear-flow situation, a unique solution exists (remarkably!) for sufficiently strong acceleration, and a one-parameter family again exists for sufficient strong deceleration.

HERB KUNZE, University of Guelph
Inverse boundary value problems in reflexive Banach spaces

In very recent work, a "collage method" for solving inverse boundary value problems has been established. The framework for the approach is built upon the Lax-Milgram theorem, cast within a Hilbert space H. In this talk, we extend both the Lax-Milgram theorem and the collage method to the setting of reflexive Banach spaces. We see that the formulation includes the earlier framework as a special case. As an example, we consider the simple boundary value problem -d/dx(K(x)du/dx)=f(x), x in [0,1], u(0)=0, u(1)=0, with f(x) in a non-Hilbertian space. We demonstrate that the new approach performs very well in solving a related inverse problem, while, not unexpectedly, the Hilbert-space based approach performs very poorly.

MARC LAFOREST, École Polytechnique de Montréal
Perturbations to non-classical shocks in non-convex systems of conservation laws

In convex systems of nonlinear conservation laws, the class of weak solutions that satisfy an entropy condition is equivalent to the class of solutions that can be obtained as limits of diffusive regularizations. This equivalence breaks down for non-convex conservation laws, like the equations of ideal magnetohydrodynamics or of certain thin film flows. In fact, the work of Bianchini and Bressan has shown that diffusive regularizations are sufficiently discriminating to allow the identification of a class of well-posed solutions even in the large family of first order hyperbolic Cauchy problems. Unfortunately, non-convex systems often appear as models with neglected high-order diffusive and dispersive physics and the previous result is insufficient.

For non-convex systems, imposing the sign AND the rate of entropy production is essentially equivalent to choosing solutions that are limits to specific diffusive-dispersive regularizations. This correspondence has been demonstrated at the level of Riemann problems but has yet to be established for non-convex conservation laws with small total variation initial data.

In this joint work with LeFloch, we exploit his theory of kinetic functions in order to establish the existence of solutions to small perturbations of non-classical shocks in systems of non-convex conservation laws, i.e. shocks that are not limits of diffusive regularizations. Given that stability and uniqueness have already been established for small total variation initial data, this work is one step towards a well-posedness theory for non-convex conservation laws, and ultimately, for hyperbolic Cauchy problems subject to general regularizations.

JEAN-PHILIPPE LESSARD, Université Laval
Rigorous Computations for Infinite Dimensional Dynamical Systems

Studying and proving the existence of solutions of nonlinear dynamical systems using standard analytic techniques is a challenging problem. In particular, this problem is even more challenging for partial differential equations, variational problems or functional delay equations which are naturally defined on infinite dimensional function spaces. As a consequence of these challenges and with the recent availability of powerful computers and sophisticated software, numerical simulations quickly became one of the primary tool used by scientists to conjecture the behaviour of the dynamics of the above mentioned nonlinear equations. A standard approach adopted by mathematicians is to get insights from numerical simulations to formulate new conjectures, and then attempt to prove the conjectures using pure mathematical techniques only. As one shall argue, this strong dichotomy need not exist in the context of dynamical systems, as the strength of numerical analysis and functional analysis can be combined to prove, in a direct computational way, existence of solutions of infinite dimensional dynamical systems. The goal of this talk is to present such rigorous numerical methods to the context of proving the existence of steady states, time periodic solutions, traveling waves and connecting orbits of finite and infinite dimensional differential equations.

EMMANUEL LORIN, Carleton University
Convergence issues of usual numerical schemes approximating 1-d nonconservative hyperbolic systems

Attempts to define weak solutions to nonconservative hyperbolic systems have lead to the development of several techniques, such as Dal Maso-LeFloch-Murat's path-based theory and the vanishing viscosity solutions described by Bianchini-Bressan. While these theories enable to define weak solutions to nonconservative hyperbolic systems, difficulties arise when numerically approximating these systems. Specifically, in the neighbourhood of a discontinuity, the numerical solutions tend to not converge to the theoretically specified weak solution of the system. In this talk we investigate some methods to numerically approximate nonconservative hyperbolic systems, we discuss why these errors arise, and we describe what weak solutions of these numerical solutions converge to. This is joint work with N. Chalmers (Waterloo).

MARK LYON, University of New Hampshire
Spectral Fourier Continuation methods for PDE Solution

The Fourier Continuation methods, which have been successfully applied to the solution of a variety of Partial Differential Equations (PDEs), allow for highly-accurate approximation and convergence in the PDE solver. Methods based on the FC(Gram) formulation are fast (FFT speed) and exhibit minimal pollution with spectral error decay away from the boundaries and a high-order polynomial interpolation based error near the boundaries. In this talk, alternative methods for applying the FC methodology will be discussed along with demonstrating algorithms that are both fast and are capable of spectral accuracy throughout the domain. The smoothing of Fourier Continuations will also be discussed.

ADAM OBERMAN, McGill
Effective approximations for nonlinear elliptic PDEs, with emphasis on the Monge-Ampère equation

Nonlinear elliptic and parabolic PDEs have applications to image processing, first arrival times in wave propagation, homogenization, mathematical finance, stochastic control and games theory. Convergent numerical schemes are important in these applications in order to capture geometric features such as folds and corners, and avoid artificial singularities which arise from bad representations of the operators.

In many cases these equations are considered too difficult to solve, which is why linearized models or other approximations are commonly used. Progress has recently been made in building solvers for a class of Geometric PDEs. I'll discuss a few important geometric PDEs which can be solved using a numerical method called Wide Stencil finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, Mean Curvature, and others.

Focusing on the Monge-Ampere equation, which is the seminal geometric PDE, I'll show how naive schemes can work well for smooth solutions, but break down in the singular case. Several groups of researchers have proposed numerical schemes which fail to converge, or converge only in the case of smooth solutions. I'll present a convergent solver which which is fast: comparable to solving the Laplace equation a few times.

The most effective notion of weak solutions for fully nonlinear elliptic equations is that of viscosity solutions, developed by Crandall, Ishii, and Lions. Viscosity solutions enjoy strong stability properties, and allow for uniform convergence of approximations, using the Barles-Souganidis theorem. This theory is used to prove convergence of the finite difference method.

The talk will be accessible to graduate students.

LISA POWERS, McGill University
Computing the Measure of Generalized Voronoi Regions

This talk introduces a fast algorithm for computing the measures of generalized Voronoi regions associated with generators of arbitrary co-dimension. The algorithm is based upon solving one Eikonal equation to generate a kernel whose iteration accumulates "mass" along the closest generator. In particular, the algorithm does not require the computation of the Voronoi diagram nor the gradient of the solution to the Eikonal equation. The algorithm is shown to be first order and converge very quickly. The method is illustrated by calculating the fraction of population living closest to each highway in the Los Angeles County highway system. This method can also be used for the fast computation of the mass centroids and higher moments of the generalized Voronoi regions.

MICHAEL RADIN, Rochester Institute of Technology
Eventually periodic and unbounded solutions of a non-autonomous max-type difference equation

We investigate the max-type difference equation in the form:

$$x[n+1] = \max\{A[n]/x[n], B[n]/x[n-1]\}$$

where $A[n]$ and $B[n]$ are periodic sequences of positive real numbers. We will study how the relationship and the rearrangement of the terms of the sequences affect the periodic solutions of the difference equation and the boundedness nature of the solutions as well. In addition, we will discuss the applications in decision making, logical gates and other related applications.

RICHARD RAND, Cornell University
Disappearance of Resonant Tongues

This work, by Rocio Ruelas and myself, investigates a phenomenon observed in systems of the form

$$dx/dt = a_1(t)\ x + a_2(t)\ y$$

$$dy/dt = a_3(t)\ x + a_4(t)\ y$$

where $a_i(t) = P_i + \epsilon\ Q_i \cos 2t$, where P_i, Q_i and ϵ are given constants,

and where it is assumed that when ϵ=0 this system exhibits a pair of linearly independent solutions of period 2π.

Since the driver $\cos 2t$ has period π, we have the ingredients for a 2:1 subharmonic resonance, which typically results in a tongue of instability involving unbounded solutions when $\epsilon > 0$. We present conditions on the coefficients P_i, Q_i such that the expected instability does not occur, i.e., the tongue of instability has disappeared.

DAVID SHIROKOFF, McGill University
A High Order Volume Penalty Method

The volume penalty method provides a simple, efficient approach for solving the incompressible Navier-Stokes equations in domains with boundaries or in the presence of moving objects. Despite the simplicity, the method suffers from poor convergence in the penalty parameter, thereby restricting accuracy of any numerical method. We demonstrate that one may achieve high order accuracy by altering the form of the penalty term. We discuss how to construct the modified penalty term, and then provide 2D numerical examples demonstrating improved convergence for the heat equation and Navier-Stokes equations. In addition, we show that modifying the penalty term does not significantly alter the time step restriction from that of the conventional penalty method.

ALEXANDRA TCHENG, McGill University
An explicit numerical method for evolving manifolds

In this talk, a new method to represent the evolution of closed codimension-1 manifolds moving with prescribed speed is proposed. First, a quick survey of the current approaches used to solve this problem is presented. Based on these, it is then shown how to devise a numerical method that keeps track of the evolving manifold explicitly, even in the case of vanishing speed. To conclude, the efficiency and accuracy of the method is illustrated with examples.

IHSAN TOPALOGLU, McGill University
An isoperimetric problem with long-range interactions related to diblock copolymers

Recently there has been interest in understanding pattern formation of diblock copolymers on the surface of a sphere. In this talk we will consider the sharp interface limit of a diblock copolymer model, and analyze the character of axisymmetric critical points defined on a two dimensional sphere. Besides being a canonical nonlocal perturbation of the isoperimetric problem, the energy functional we are going to investigate also poses challenges as a model of energy-driven pattern formation caused by competing short- and long-range interactions. This is a joint work with Rustum Choksi and Gantumur Tsogtgerel.

JOZSEF VASS, University of Waterloo
An Exact Model of Fully Developed Turbulence

Fully developed turbulence occurs at the infinite extreme of the Reynolds spectrum. It is a theoretical phenomenon which can only be approximated experimentally or computationally, and thus its precise properties are only hypothetical, though widely accepted. It is considered to be a chaotic yet stationary flow field, with self-similar fractalline features. A number of approximate models exist, often exploiting this self-similarity. We hereby present the mathematical model of fractal potential flows, and link it philosophically to the phenomenon of turbulence, building on the experimental observations of others. The model hinges on the recursive iteration of a fluid dynamical transfer operator. We show the existence of a unique attractor in an appropriate space, which will pose as our model for the fully developed turbulent flow field. Its singularities are shown to form an IFS fractal, resolving Mandelbrot's conjecture. Meanwhile we present an isometric isomorphism between flows and probability measures, hinting at a wealth of future research.

TAMAS WIANDT, RIT
Bifurcation scenarios in a system of delay differential equations

A system of two identical, delay coupled semiconductor lasers is considered. We investigate the geometric background of the existence of compound laser mode (CLM) solutions and describe the bifurcation scenarios depending upon the coupling rate. The structure of stable CLM solutions is characterized on the coupling rate - detuning parameter domain.

MICHAEL YAMPOLSKY,