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Effective approximations for nonlinear elliptic PDEs, with emphasis on the Monge-Ampère equation

Nonlinear elliptic and parabolic PDEs have applications to image processing, first arrival times in wave propagation, homogenization, mathematical finance, stochastic control and games theory. Convergent numerical schemes are important in these applications in order to capture geometric features such as folds and corners, and avoid artificial singularities which arise from bad representations of the operators.

In many cases these equations are considered too difficult to solve, which is why linearized models or other approximations are commonly used. Progress has recently been made in building solvers for a class of Geometric PDEs. I'll discuss a few important geometric PDEs which can be solved using a numerical method called Wide Stencil finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, Mean Curvature, and others.

Focusing on the Monge-Ampere equation, which is the seminal geometric PDE, I'll show how naive schemes can work well for smooth solutions, but break down in the singular case. Several groups of researchers have proposed numerical schemes which fail to converge, or converge only in the case of smooth solutions. I'll present a convergent solver which which is fast: comparable to solving the Laplace equation a few times.

The most effective notion of weak solutions for fully nonlinear elliptic equations is that of viscosity solutions, developed by Crandall, Ishii, and Lions. Viscosity solutions enjoy strong stability properties, and allow for uniform convergence of approximations, using the Barles-Souganidis theorem. This theory is used to prove convergence of the finite difference method.

The talk will be accessible to graduate students.