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Transcending ω_1-sequences of reals

I will describe a procedure which, to each ω_1-sequence of reals, assigns a sequence of tree orderings on ω_1 which attempts to build a real number not in the range of the sequence. If this attempt fails, the result is a tree of countable closed subsets of ω_1 which has no uncountable branch, is completely proper as a forcing notion (and remains so in any outer model with the same reals in which T does not have an uncountable branch), and is "self specializing" in the sense that

$$\{(s,t) \in T^2 : (\mathrm{ht}(s) = \mathrm{ht}(t)) \wedge (s \neq t)\}$$

can be decomposed into countably many antichains. In particular, this tree can have at most one branch in any outer model. This in particular shows that the forcing axiom for completely proper forcings is inconsistent with the Continuum Hypothesis, thus answering a longstanding problem of Shelah.