Representations of Algebras
Représentations d'algèbres
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LUCHEZAR L. AVRAMOV, University of Nebraska - Lincoln
Cohomologically noetherian modules and rings

Every left module M over an associative ring R defines functors $\mathrm{Ext}^*_R(M,-)$ and $\mathrm{Ext}^*_R(-,M)$ from left R-modules to graded right $\mathrm{Ext}^*_R(M,M)$-modules and to graded right $\mathrm{Ext}^*_R(M,M)$-modules, respectively. Over various classes of rings, including group algebras over fields and complete intersection local rings, it is known that these functors map noetherian R-modules to modules that are noetherian over the universal center of $\mathrm{Ext}^*_R(M,M)$. It will be shown that such a condition on cohomology implies strong restrictions on the ring R. The talk is based on joint work with Srikanth Iyengar.

YURI BAHTURIN, Memorial University of Newfoundland
Group gradings on locally finite simple algebras

We study gradings by groups on locally finite associative and Lie algebras. In a number of cases we classify such gradings up to isomorphism of graded algebras. These cases include simple associative and Lie algebras of finitary linear transformations of not necessarily finite-dimensional vector spaces. These results are contained in a joint paper with Matej Brešar and Mikhail Kotchetov (arxiv: 1106.2638).

MARGARET BEATTIE, Mount Allison University
Twistings of Hopf algebras whose coradical is a sub-Hopf algebra

Let A be a Hopf algebra over a field of characteristic 0 with coradical H such that H is a finite dimensional sub-Hopf algebra of A. Then H is a semisimple Hopf algebra so that there is a total integral $\lambda \in H^*$ and λ is left H-linear with respect to the adjoint action of H on itself. Then there is an H-bilinear coalgebra projection π from A to H. If π is a bialgebra map, then $A \cong R\#H$, the Radford biproduct or bosonization of H with $R := A^{co\pi}$, the algebra of coinvariants. Here R is a connected Hopf algebra in the category ${}^H_H\mathcal{YD}$.

If π does not preserve multiplication, then $A \cong R\#_\xi H$ where R is a pre-bialgebra in ${}^H_H\mathcal{YD}$ and $\xi : R \otimes R \to H$. The question is whether A can be twisted by a cocycle to a Radford biproduct. We show that the correct setting for this problem is that of dual quasi-bialgebras and that A can always be twisted by a gauge transformation to a bosonization $Q\#H$ where Q is a connected dual quasi-bialgebra in ${}^H_H\mathcal{YD}$. This work is joint with A. Ardizzoni and C. Menini.

FRAUKE BLEHER, University of Iowa
Automorphisms of Katz-Gabber covers

This is joint work with Ted Chinburg, Bjorn Poonen and Peter Symonds. Let k be an algebraically closed field of characteristic $p > 0$, and let G be a finite group of order divisible by p. This talk has to do with the possible isomorphism classes of G-sets T which can arise as the ramification locus of a faithful action of G on a smooth projective curve X over k. One says this action defines a Katz-Gabber G-cover if X/G has genus 0, T^G is one point, and G acts tamely and transitively on $T - T^G$. Our goal is to study the automorphism group $\mathrm{Aut}_k(X)$ of X over k. This group contains G by virtue of the Katz-Gabber construction.

We consider the case when $X \to X/\mathrm{Aut}_k(X)$ is not a Katz-Gabber cover for the group $\mathrm{Aut}_k(X)$, so that $\mathrm{Aut}_k(X)$ is strictly larger than G. We show that then either X belongs to an exceptional family of curves of genus 0 or 1, or the action of $\mathrm{Aut}_k(X)$ on X mixes the tame and totally ramified orbits. The exceptional case in which X has genus 1 is of particular interest because it leads to some new explicit formulas for power series $\sigma(t) = \sum_{i=1}^{\infty} a_i t^i$ which define automorphisms of $k[[t]]$ of order p^2 when $p = 2$.

THOMAS BRÜSTLE, Université de Sherbrooke and Bishop's University
Projective dimension of modules over cluster-tilted algebras

In joint work with Louis Beaudet and Gordana Todorov we study the projective dimension of modules over a cluster-tilted algebra End (T) where T is a tilting object in a cluster category $\mathcal{C}$. It is well-known that all modules are of the form Hom(T, M) for some object M in $\mathcal{C}$, and since End(T) is Gorenstein of dimension 1, the projective dimension of Hom(T, M) is either zero, one or infinity.

We consider the ideal I_M of End$_{\mathcal{C}}(T[1])$ given by all endomorphisms that factor through M, and show that the End$(T)-$module Hom(T, M) has infinite projective dimension precisely when I_M is non-zero. Examples indicate that the objects M of $\mathcal{C}$ such that $I_M \neq 0$ lie on hammocks in $\mathcal{C}$.

XUEQING CHEN, University of Wisconsin-Whitewater
Integral bases of quantum cluster algebras for affine valued quivers

Let Q be an acyclic valued quiver. Recently, Rupel proved the quantum Caldero-Chapoton formula which gives a bijection from the set of indecomposable rigid valued representations of Q to the set of non-initial quantum cluster variables for the corresponding quantum cluster algebra. This correspondence enables us to obtain integral bases of the quantum cluster algebras for affine valued quivers by using the standard monomials constructed by Berenstein and Zelevinsky in their paper "Quantum cluster algebras". This talk is based on a joint work with M. Ding and J. Sheng.

ED GREEN, Virginia Tech
Group gradings and actions on Brauer graph algebras

I will discuss results of joint work with Sybille Schroll and Nicole Snashall of the University of Leicester, England. I begin by recalling the definitions of Brauer graphs and Brauer graph algebras. Group actions and gradings on Brauer graphs are introduced and it is shown how these are related to group actions and gradings on Brauer graph algebras. Some applications are mentioned.

BIRGE HUISGEN-ZIMMERMANN, University of California at Santa Barbara
Generic Representation Theory of Quivers with Relations

We address the irreducible components of the algebraic varieties parametrizing classes of finite dimensional representations of a finite dimensional algebra, in terms of both their geometry and the generic properties of the modules they encode. In this talk, we mainly focus on the second theme, sketching some background and following with joint results of the speaker, Eric Babson and Rekha Thomas.

KIYOSHI IGUSA, Brandeis University
Continuous cluster functions

A cluster function is a homomorphism from the semi-ring of Laurent polynomials in cluster variables to the semi-ring of positive real numbers. Since continuous cluster categories have a topology on the set of objects, we can ask for a continuous cluster function. We will use the formula of Assem, Dupont, Schiffler and Smith to examine limits of cluster functions on infinite string modules to obtain conditions on the initial seed which will generate a continuous cluster function. This is joint work with Gordana Todorov.

COLIN INGALLS, University of New Brunswick
Noncommutative surfaces and curves of finite representation type

This is joint work with Daniel Chan. Local orders of global dimension two, over surfaces of finite representation type have been classified geometrically by Artin and by AR quivers by Reiten and Van den Bergh. We present a third classification via central extensions of finite subgroups of GL_2. This methods easily allows one to link all three classifications. We further classify noncommutative curves of finite representation type using noncommutative matrix factorizations and the classification of orders.

ELLEN KIRKMAN, Wake Forest University
Invariant subrings of $\mathbb{C}_{-1}[x_1, \ldots, x_n]$ under permutation actions

Let $A = \mathbb{C}_{-1}[x_1, \ldots, x_n]$ be the skew polynomial algebra $x_j x_i = -x_i x_j$ for all $i \neq j$. The symmetric group S_n acts on A by permuting the indices. Let G be a subgroup of S_n. The subring of invariants A^G is an Artin-Schelter Gorenstein algebra. We compare properties of A^G to those of invariants of the commutative polynomial ring $\mathbb{C}[x_1 \ldots, x_n]^G$. (With James Kuzmanovich and James Zhang)

MARK KLEINER, Syracuse University
Induction and restriction functors in representation theory of partially ordered sets

Given a finite group with a subgroup, the induction and restriction functors are useful for understanding the structure of representations of the group. Given a finite partially ordered set with a subset, one defines the induction, coinduction, and restriction functors in a natural way. We discuss how one can use these functors to get information about the category of representations of the partially ordered set.

GRAHAM LEUSCHKE, Syracuse University
Non-commutative desingularizations of determinantal varieties

This talk is about joint work with R.-O. Buchweitz and M. Van den Bergh. We construct non-commutative resolutions of determinantal varieties defined by arbitrary-size minors of a generic matrix. The resolution can be presented as the path algebra of a quiver with relations, where the underlying graph of the quiver is an ideal in the Young lattice of partitions.

FERO MARKO, The Pennsylvania State University
Irreducibility of induced modules for general linear supergroups

We determine when is an induced module $H^0_G(\lambda)$, corresponding to the highest weight λ and the general linear supergroup $G = GL(m|n)$ irreducible. The answer uses the description of irreducible Specht modules given by Fayers and a characterization of induced G-modules given by Zubkov. In the (easier) case of characteristic zero this provides the converse of the factorization formula for hook Schur functions given by Berele and Regev.

ROBERTO MARTINEZ-VILLA, Centro de Investigacion en Matematicas, UNAM
Tilting and functor categories.

We will report joint results with Martin Ortiz-Morales on tilting in functor categories. The aim is to extend results on artin algebras, like classical tilting and Happel's theorem, to functor categories.

ALEX MARTSINKOVSKY, Northeastern University
Stabilizing the tensor product

We introduce the concept of asymptotic stabilization of the tensor product and construct a comparison map from Vogel homology to the asymptotic stabilization. This is joint work with Jeremy Russell.

PUIMAN NG, Université de Sherbrooke
Torsion theories in the cluster category of Dynkin type A_∞

Let $\mathcal{D}$ be the cluster category of Dynkin type A_∞ with translation functor Σ. Let $\mathcal{U}$ be a cluster tilting subcategory of $\mathcal{D}$. Then by definition ($\mathcal{U}$, $\Sigma\mathcal{U}$) is a torsion theory. Now given any subcategory $\mathcal{X}$ of $\mathcal{D}$, we provide the equivalent conditions for the pair $(\mathcal{X}, \mathcal{Y})$ to be a torsion theory for some subcategory $\mathcal{Y}$ of $\mathcal{D}$.

Given any subcategory $\mathcal{U}$ of $\mathcal{D}$, not all torsion theories $(\mathcal{U},\mathcal{U}^\perp)$ are t-structures. Therefore we press on to give a characterization of t-structures in $\mathcal{D}$ on the Auslander-Reiten quiver. The simplicity of the way t-structures are visualized on the Auslander-Reiten quiver gives a subtle hint with regard to the description of the "t-structure triangles" in $\mathcal{D}$, which turns out to be faithful to our intuition by a rigorous proof with the involvement of the octahedral axiom.

CHARLES PAQUETTE, University of New Brunswick
Some results on the Auslander-Reiten theory of infinite quivers

Let k be a field and Q be a strongly locally finite quiver, that is, Q is a locally finite quiver such that for any pair of vertices x, y, there are finitely many paths from x to y. In a joint work with R. Bautista and S. Liu, we get a complete description of the Auslander-Reiten quiver of $\mathrm{rep}^+(Q)$, the category of the finitely presented representations of Q over k. In this talk, I will discuss these results and extend them to get a partial description of the Auslander-Reiten quiver of the whole category $\mathrm{rep}(Q)$ of the locally finite dimensional representations of Q over k. To this end, I will introduce a full subcategory of $\mathrm{rep}(Q)$ which contains most of the Auslander-Reiten theory of $\mathrm{rep}(Q)$.

MARJU PURIN, Manhattan College
The Generalized Auslander-Reiten Conjecture and Derived Equivalences

This is joint work with Kosmas Diveris. In a paper from 1975, Maurice Auslander and Idun Reiten stated the following conjecture: "If M is an R-module with $\mathrm{Ext}^i(M, M \oplus R) = 0$ for all $i > 0$, then M is a projective module." This conjecture remains open for "most" classes of rings, including for artin algebras and commutative rings. A natural generalization is the following statement: "If M is an R-module with $\mathrm{Ext}^i(M, M \oplus R) = 0$ for all $i >> 0$, then M has finite projective dimension." In our talk we discuss these conjectures, give a version of the latter conjecture for any triangulated category, and use it to show that the generalized version of the Auslander-Reiten Conjecture is stable under any derived equivalence of noetherian rings.

ALISTAIR SAVAGE, University of Ottawa
Classification of irreducible quasifinite modules over map Virasoro algebras

The Virasoro algebra plays a fundamental role in the theory of vertex operator algebras, conformal field theory, string theory, and the representation theory of affine Lie algebras. In this talk we will present a recent classification of the irreducible quasifinite modules (that is, modules with finite-dimensional weight spaces) for map Virasoro algebras (tensor products of the Virasoro algebra with a Noetherian commutative associative algebra). We will see that all such modules are tensor products of generalized evaluation modules. We also give an explicit sufficient (and, in some cases, necessary) condition for a Verma module of a map Virasoro algebra to be irreducible.

BENJAMIN STEINBERG, City College of New York
Combinatorial topology and the global dimension of left regular bands

Work of Bidigare, Hanlon, Rockmore, Diaconis, Brown and others has shown that a large number of combinatorial objects have the structure of a kind of monoid known as a left regular band (LRB). Examples include hyperplane arrangements, oriented matroids and matroids. The representation theory of LRBs can be used to analyze random walks on these objects including

classical Markov chains like card-shuffling. LRBs also play a key role in understanding Solomon's descent algebra associated to a finite Coxeter group.

LRB algebras are directed quasi-hereditary algebras and hence have finite global dimension. In this talk, we discuss how to use combinatorial topology to compute the global dimension of an LRB algebra and in fact arbitrary Ext between simple modules. In particular we show that the Leray number of a flag complex is precisely the global dimension of an associated LRB algebra. Our main technical tools are classifying spaces of small categories and Quillen's Theorem A.

This is joint work with Stuart Margolis and Franco Saliola.

HUGH THOMAS, University of New Brunswick

Quotient-closed subcategories of the representations of a quiver

Let Q be a quiver without oriented cycles. We show that the quotient-closed full subcategories of rep Q which are cofinite (i.e. contain all but finitely many indecomposable representations of Q) are naturally in bijection with the elements of the Weyl group W associated to Q, and that the inclusion order on the subcategories corresponds to the "sorting order" on W introduced by Armstrong. In the Dynkin case, cofiniteness is trivially satisfied, and we obtain a bijection between the quotient-closed subcategories and the elements of W. Even the fact that these sets have the same cardinality seems to be new. The representation theory of preprojective algebras plays an important role in our analysis. This is joint work with Steffen Oppermann and Idun Reiten.

GORDANA TODOROV, Northeastern University

From Frobenius categories to Cluster Structures

By the result of Happel it is known that the stable categories of Frobe- nius categories are triangulated. We discuss situations in which such stable cat- egories have cluster tilting objects, cluster structure, or are triangle-equivalent to generalized cluster categories associated to algebras of global dimension 2.

The well known such classes of Frobenius categories are the subcategories of finitely presented modules over preprojective algebras associated to the elements in Coxeter groups as done in [GLS] and [BIRSc]. The stable categories of these categories have cluster tilting objects and cluster structure. It was shown in [AIRT] that certain Frobenius categories with cluster tilting objects are triangle equivalent to generalized cluster categories associated to algebras of global dimension 2. In particular it is true for the above categories associated to the elements of Coxeter groups. Another interesting Frobenius category is the category of representations of circle, with the stable category being the continuous cluster category [IT].

HELENE TYLER, Manhattan College

Auslander-Reiten Layers in the Rhombic Picture of an Algebra

In this joint work with Markus Schmidmeier, we decompose the rhombic picture of a hereditary algebra into layers corresponding to the preprojective, regular, and preinjective components of the Auslander-Reiten quiver. The rhombic picture, which was defined by Ringel, uses the Gabriel-Roiter measure and Gabriel-Roiter comeasure to define the position of a module. We show that certain families of modules in the Auslander-Reiten components give rise to limit points in the rhombic picture. Illustrative examples will be presented via quivers of type $\tilde{\mathbb{A}}_n$.

DAN ZACHARIA, Syracuse University

On rigid sheaves over $\mathbb{P}^n$.

I will talk on joint work with Dieter Happel. We prove that if E is an indecomposable coherent sheaf over the projective n-space such that $\mathrm{Ext}^1(E,E)=0$, then E has a trivial endomorphism ring. This generalizes a result of Drézet for rigid sheaves over $\mathbb{P}^2$. The proof involves reduction to rigid modules over a finite dimensional algebra of Loewy length 2 using the Koszul algebra structure of the polynomial ring.