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Combinatorial topology and the global dimension of left regular bands

Work of Bidigare, Hanlon, Rockmore, Diaconis, Brown and others has shown that a large number of combinatorial objects have the structure of a kind of monoid known as a left regular band (LRB). Examples include hyperplane arrangements, oriented matroids and matroids. The representation theory of LRBs can be used to analyze random walks on these objects including classical Markov chains like card-shuffling. LRBs also play a key role in understanding Solomon's descent algebra associated to a finite Coxeter group.

LRB algebras are directed quasi-hereditary algebras and hence have finite global dimension. In this talk, we discuss how to use combinatorial topology to compute the global dimension of an LRB algebra and in fact arbitrary Ext between simple modules. In particular we show that the Leray number of a flag complex is precisely the global dimension of an associated LRB algebra. Our main technical tools are classifying spaces of small categories and Quillen's Theorem A.

This is joint work with Stuart Margolis and Franco Saliola.