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The quantum query complexity of read-many formulas

The quantum query complexity of evaluating any read-once formula with n black-box input bits is $\Theta(\sqrt{n})$. However, the corresponding problem for read-many formulas (i.e., formulas in which the inputs have fanout) is not well understood. Although the optimal read-once formula evaluation algorithm can be applied to any formula, it can be suboptimal if the inputs have large fanout. We give an algorithm for evaluating any formula with n inputs, size S, and G gates using $O(\min\{n, \sqrt{S}, n^{1/2}G^{1/4}\})$ quantum queries. Furthermore, we show that this algorithm is optimal, since for any n, S, G there exists a formula with n inputs, size at most S, and at most G gates that requires $\Omega(\min\{n, \sqrt{S}, n^{1/2}G^{1/4}\})$ queries. We also show that the algorithm remains nearly optimal for circuits of any particular depth of at least 3, and we give a linear-size circuit of depth 2 that requires $\Omega(n^{0.555})$ queries. Applications of these results include an $\Omega(n^{1.055})$ lower bound for Boolean matrix product verification, a nearly tight characterization of the quantum query complexity of evaluating constant-depth circuits with bounded fanout, new formula gate count lower bounds for several functions including parity, and a construction of an AC^0 circuit of linear size that can only be evaluated by a formula with $\Omega(n^{2-\epsilon})$ gates.

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