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**Plenary Speakers**  
**Conférences plénières**

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**HERMANN EBERL**, University of Guelph

*The Good, the Bad, and the Ugly: How mathematics helped to explore the fantastical world of biofilms*

Bacterial biofilms are microbial depositions on submerged interfaces. Bacteria attach to the surface and start producing a slimy substance that protects them. While many environmental engineering techniques are based on the good aspects of biofilms, they are harmful in medical and industrial contexts. The term “biofilm”, however, is a misnomer: Rather than as a homogeneous film, they often develop in complex spatially heterogeneous structures and morphologies. For a decade now, modeling how these structures arise has increasingly drawn mathematical attention (not to mention the numerous cellular automata models or individual based models that have been developed for an only slightly longer period by engineers and biologists). Bacterial biofilms can be understood both as spatially structured populations and as complex fluids. Taking, independently, either view as a starting point we can derive a biofilm model in the form of a quasilinear parabolic equation that simultaneously comprises two non-linear diffusion effects: a porous medium equation degeneracy as the dependent variable vanishes and a super-diffusion singularity as it approaches an a priori known upper bound. In this talk we will focus on the mathematical results for this type of equation and show some numerical simulations. Time permitting (i.e. likely not) we will also discuss an extension of the model to include quorum sensing as a means of bacterial communication.

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**CHRISTINA GOLDSCHMIDT**, University of Oxford

*The scaling limit of the critical random graph*

The basic object in this talk will be the celebrated Erdős-Rényi random graph model: we take  $n$  labelled vertices, any pair of which we connect independently with probability  $p$ . A fundamental fact about this model is that it undergoes a phase transition. Set  $p = c/n$ , where  $c$  is a constant. Then for  $c < 1$ , the components of the graph have size which is at most of order  $\log n$ ; for  $c > 1$ , there is a single *giant* component of size order  $n$  and all other components again have size at most of order  $\log n$ . In order to investigate the critical behaviour, it is useful to look inside the *critical window*, where  $p = 1/n + \lambda n^{-4/3}$  and  $\lambda \in \mathbb{R}$ . Here, it turns out that the largest components all have size  $n^{2/3}$ . Moreover, they are close to being trees, in that each differs from a tree by a number of edges which stays bounded in expectation as  $n \rightarrow \infty$ . Viewing the components as random metric spaces in which we rescale the graph distance by  $n^{-1/3}$ , we are able to give a complete description of the scaling limit of the critical random graph. The limiting object has a satisfyingly elegant description as a sequence of continuum random trees in each of which a finite number of random vertex-identifications has been made.

This is joint work with Louigi Addario-Berry (McGill) and Nicolas Broutin (INRIA Paris-Rocquencourt).

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**GORDON SWATERS**, University of Alberta

*Mathematical properties of a higher-order model for nonlinear internal waves*

Recent theoretical advances in connecting the wave-induced mean flow with the conserved pseudomomentum per unit mass has permitted the first rational derivation of a model that describes the weakly nonlinear propagation of internal gravity plane waves in a continuously stratified fluid. Depending on the particular parameter regime examined, the new model corresponds to an extended bright or dark derivative nonlinear Schrödinger equation or an extended complex-valued modified Korteweg-de Vries or Sasa-Satsuma equation. Here, we present the mass, momentum and energy conservation laws. A noncanonical and nonlocal infinite-dimensional Hamiltonian formulation of the model is introduced. The modulational stability characteristics associated with the Stokes wave solution of the model are described. The bright and dark solitary wave solutions of the model are given.

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**CRAIG TRACY**, University of California, Davis

*Turbulent liquid crystals, KPZ universality and the asymmetric simple exclusion process*

We review recent experimental work on stochastically growing interfaces and compare these results with the recent theoretical developments for the Kardar-Parisi-Zhang equation and the closely related asymmetric simple exclusion process. The emphasis in this talk will be on the underlying results in the asymmetric simple exclusion process.

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**HUGH WOODIN**, UC Berkeley  
*Towards an ultimate version of  $L$*

There is now evidence that there is an "ultimate" version of Gödel's constructible universe  $L$ . The axiom that  $V$  is this ultimate  $L$  will yield a conception of the universe of sets which is both compatible with all known axioms of infinity (which are consistent with the Axiom of Choice) and which resolves essentially all the problems which have been shown to be unsolvable on the basis of the ZFC axioms using Cohen's method of forcing.

But evidence can be misleading and perhaps this evidence is simply a prelude to a refutation that there is an ultimate version of  $L$ . I will survey the situation and the relevant conjectures.