Operator Algebras
Algèbres d'opérateurs
(Org: **Pinar Colak** (SFU), **George Elliott** (Toronto), **Zhiqiang Li** (Toronto), **Henning Petzka** (Toronto), **Adam Sierakowski** (Toronto) and/et **Aaron Tikuisis** (Toronto))

MICHAEL BRANNAN, Queen's University
On the von Neumann algebras associated to quantum permutation groups.

In 1998, Shuzhou Wang showed that the permutation group S_N (acting on the finite set $X_N = \{1, 2, \ldots, N\}$) admits a natural analogue within the category of *quantum* transformation groups acting on X_N. The resulting compact quantum group is denoted by S_N^+, and is called the quantum permutation group.

In this talk, we will study the reduced von Neumann algebra $L^\infty(S_N^+)$ associated to the quantum group S_N^+. Unlike the situation for S_N (where of course $L^\infty(S_N) = C(S_N) \cong \mathbb{C}^{N!}$), $L^\infty(S_N^+)$ turns out to be a non-injective finite von Neumann algebra as soon as $N \geq 5$. We prove that $L^\infty(S_N^+)$ always has the Haagerup property and is a full type II_1-factor when $N \geq 8$.

NATE BROWN, Penn State
Dynamical Systems Associated to II_1-factors

In joint work with Valerio Capraro, we have recently found some interesting Hom-space type invariants that can be associated to finite factors. I'll describe the construction and mention some of the main results so far.

MAN-DUEN CHOI, University of Toronto
The symmetric structure of the Cuntz Algebra on two generators

We consider the Cuntz C*-algebra generated by two isometries whose range spaces are orthogonally complemented. This algebra plays an important role in structure theory, for many spectacular features due to the symmetry of two canonical isometries. In particular, we show that the C*algebra is *-isomorphic to both of its fixed point algebra and the crossed product algebra , with respect to the automorphism of interchanging of two generators. This research is a joint work with Frederic Latremoliere of University of Denver.

NHAN-PHU CHUNG, SUNY at Buffalo, NY, USA
Entropy, homoclinic group, algebraic actions and von Neumann algebras

Homoclinic points describe the asymptotic behavior of group actions on spaces and play an important role in the general theory of dynamical systems. In 1999, Doug Lind and Klaus Schmidt established relations between homoclinic points and entropy properties for expansive algebraic actions of $\mathbb{Z}^d$. Their proof depends heavily on the commutative factorial Noetherian ring structure of the integral group ring of $\mathbb{Z}^d$.

In a joint work with Hanfeng Li, we extend their results to expansive algebraic actions of polycyclic-by-finite groups.

Applying our results to the field of von Neumann algebras, we get a positive answer to a question of Deninger about the Fuglede-Kadison determinant to the case group is amenable. We also prove that for an amenable group, an element in the integral group ring is a non-zero divisor if and only if the entropy of corresponding principal algebraic action is finite.

KEN DAVIDSON, University of Waterloo
Dilation theory, commutant lifting and semicrossed products

We take a new look at dilation theory for nonself-adjoint operator algebras. Among the extremal (co)extensions of a representation, there is a special property of being fully extremal. This allows a refinement of some of the classical notions which

are important when one moves away from standard examples. This leads to variations of the notions of commutant lifting and Ando's theorem. This is applied to the study of semicrossed products by automorphisms, and endomorphisms which lift to the C*-envelope. (This is joint work with Elias Katsoulis.)

ILIJAS FARAH, York University
Group cohomology and automorphisms of corona algebras

I will exploit a connection between the derived inverse limit functors $\varprojlim^{(n)}$ and the structure of automorphism groups of corona algebras. By combining this with some simple ideas from set theory I will prove that the Continuum Hypothesis implies that for every stable separable C*-algebra A its corona C(A) has many outer automorphisms. This is a joint work with Sam Coskey.

FARZAD FATHIZADEH, York University
Scalar Curvature for the Noncommutative Two Torus II

We give a local expression for the $scalar\ curvature$ of the noncommutative two torus $A_\theta = C(\mathbb{T}^2_\theta)$ equipped with an arbitrary translation invariant complex structure and Weyl conformal factor. The metric information is encoded in the Dirac operator D of a $twisted\ spectral\ triple$ on A_θ so that we view this C^*-algebra as a noncommutative Riemannian manifold. The local expression for curvature is computed by evaluating the value of the (analytic continuation of the) $spectral\ zeta\ function$

$$\zeta_a(s) := \mathsf{Trace}(a|D|^{-s})$$

at $s = 0$ as a linear functional in $a \in C^\infty(\mathbb{T}^2_\theta)$. A new, purely noncommutative, feature here is the appearance of the $modular\ automorphism\ group$ from the theory of type III factors and quantum statistical mechanics in the final formula for the curvature. This formula coincides with the formula that was recently obtained independently by Connes and Moscovici in their recent paper. At the end, we will explain how this formula fits into our earlier work on $Gauss\text{-}Bonnet\ theorem$ for noncommutative two tori, which extends the Gauss-Bonnet theorem of Connes and Tretkoff to general conformal structures on $\mathbb{T}^2_\theta$. This is joint work with Masoud Khalkhali.

ADAM FULLER, University of Waterloo
Semicrossed Product Algebras

In this talk we will discuss nonself-adjoint crossed product algebras formed by the action of a semigroup $\mathcal{S}$ by endomorphisms $\{\alpha_s\}_{s\in\mathcal{S}}$ on a unital operator algebra $\mathcal{A}$. In particular we will be looking at crossed products by semigroups of the form $\mathcal{S} = \sum_{i=1}^{\oplus k} \mathcal{S}_i$ where each $\mathcal{S}_i$ is a subsemigroup of the positive real line. We restrict ourselves to the the crossed product algebras which relate to what are known as Nica-covariant (aka doubly-commuting) representations of the dynamical system. This restriction is partly due to the impossibility of forming a dilation theory for more general representations. We will conclude with a discussion of the C^*-envelope of these algebras.

THIERRY GIORDANO, University of Ottawa
On type III representations of simple nuclear C-algebras.*

In 1967, Powers proved that two representations π_1 and π_2 (on a separable Hilbert space) of a UHF algebra A are algebraically equivalent iff there is an automorphism α of A such that $\pi_1 \circ \alpha$ and π_2 are quasi-equivalent. In this talk I will present the extension of this result to simple separable nuclear C*-algebras when the representations are of type III. This result solves a question of A. Kishimoto and is a joint work with Ping Ng.

RYAN HAMILTON, University of Waterloo
Pick interpolation and operator algebras

Suppose $z_1, \ldots, z_n$ are points in the complex unit disk and $w_1, \ldots, w_n$ are complex numbers. The Nevanlinna-Pick interpolation theorem of the early 20th century states that there is a bounded analytic function f on the disk satisfying $f(z_i) = w_i$ for $i = 1, \ldots, n$ and $\|f\|_\infty \leq 1$ if and only if the matrix

$$\left[\frac{1 - w_i\overline{w_j}}{1 - z_i\overline{z_j}}\right]_{i,j=1}^{n}$$

is positive semidefinite. In general, the analogue of the Nevanlinna-Pick theorem fails to hold for other domains. This talk will discuss a recent treatment of this type of problem using operator algebraic techniques. A general Pick-type theorem for domains in $\mathbb{C}^d$ will be presented.

VADIM KAIMANOVICH, University of Ottawa
Finite approximations of invariant measures

We show that any invariant measure on the space of rooted graphs can be approximated by uniform measures on finite graphs provided the root moving equivalence relation is amenable with respect to this measure.

EVGENIOS KAKARIADIS, University of Waterloo
Dilations of C-correspondences*

Motivated from semicrossed products, we revisit dilations of a non-injective C*-correspondence to an essential Hilbert bimodule, under the constraint that their corresponding Cuntz-Pimsner algebras are Morita equivalent. We show how this can be achieved by extending the technique of adding tails established by Muhly and Tomforde. Several examples show the necessity for this extension as our technique provides a control on the classes of the tensor algebras and their C*-envelopes. For example, using this construction we are able to prove that the C*- envelope of a semicrossed product of a dynamical system is Morita equivalent to a crossed product (joint work with E. Katsoulis).

MATTHEW KENNEDY, Carleton University
The structure of an isometric tuple

The definition of an isometric tuple is a natural higher-dimensional generalization of the definition of an isometry. An n-tuple of operators $V = (V_1, ..., V_n)$ acting on a Hilbert space is said to be isometric if the row operator $[V_1, ..., V_n]$ is an isometry. The classical Lebesgue-Wold decomposition of an isometry tells us that an isometry can be written as the direct sum of a unilateral shift, an absolutely continuous unitary and a singular unitary. In this talk, we will discuss a higher-dimensional generalization of this decomposition for an isometric tuple. As in the classical case, this decomposition determines the weakly closed algebra and the von Neumann algebra generated by the tuple.

DAVID KERR, Texas A&M University
Sofic measure entropy via finite partitions

Recently Lewis Bowen introduced a notion of entropy for measure-preserving actions of sofic groups and used it to obtain a far-reaching extension of the Ornstein-Weiss classification of Bernoulli actions over amenable groups. Subsequently Hanfeng Li and I developed a more general operator-algebraic approach to sofic entropy and established a variational principle in this context. I will show that these two perspectives can be reconciled to produce a definition with the novelty that it does not depend on generators, like the standard formulation of classical measure entropy due to Sinai. This leads in particular to a streamlined computation for Bernoulli actions, as I will describe.

MASOUD KHALKHALI, Western Ontario
Scalar Curvature for the Noncommutative Two Torus I

We give a local expression for the *scalar curvature* of the noncommutative two torus $A_\theta = C(\mathbb{T}^2_\theta)$ equipped with an arbitrary translation invariant complex structure and Weyl conformal factor. The metric information is encoded in the Dirac operator D of a *twisted spectral triple* on A_θ so that we view this C^*-algebra as a noncommutative Riemannian manifold. The local expression for curvature is computed by evaluating the value of the (analytic continuation of the) *spectral zeta function*

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at $s = 0$ as a linear functional in $a \in C^\infty(\mathbb{T}^2_\theta)$. A new, purely noncommutative, feature here is the appearance of the *modular automorphism group* from the theory of type III factors and quantum statistical mechanics in the final formula for the curvature. This formula coincides with the formula that was recently obtained independently by Connes and Moscovici in their recent paper. At the end, we will explain how this formula fits into our earlier work on *Gauss-Bonnet theorem* for noncommutative two tori, which extends the Gauss-Bonnet theorem of Connes and Tretkoff to general conformal structures on $\mathbb{T}^2_\theta$. This is joint work with Farzad Fathizadeh.

MARCELO LACA, Victoria

C-algebras of Toeplitz type from number fields*

We study the Toeplitz-type C*-algebra $\mathcal{T}[R_K]$ generated by the left regular representation of the affine semigroup of the ring of integers R_K in an algebraic number field K. We give a presentation in terms of generators and relations and describe $\mathcal{T}[R_K]$ as a semigroup crossed product. We also show that the correspondence $K \to \mathcal{T}[R_K]$ is functorial with respect to field extensions, and we compute the KMS equilibrium states of a natural time evolution, which exhibit a phase transition associated to the ideal class group of K. This is joint work with J. Cuntz and C. Deninger.

GREG MALONEY, University of Massachusetts Boston

Computing the K-theory of one-dimensional mixed substitution tiling spaces

To any substitution tiling space that satisfies certain conditions, one can associate a C*-algebra. Using a topological approximant to the tiling space called the Anderson-Putnam complex, it is possible to compute the K_0 group of this C*-algebra. The goal of this work is to generalize these results to tiling spaces that arise from not just one, but several different substitutions, all acting on the same set of tiles. Our results are restricted to one-dimensional tiling spaces. This is joint work with Franz Gähler.

JAMES MINGO, Queen's University

Second Order Even and R-diagonal Operators

Voiculescu showed that circular operators, the free analogue of complex Gaussian random variables, and Haar unitaries are related by polar decomposition. This relationship was deepened by Nica and Speicher who showed that both are examples of R-diagonal operators. Nica and Speicher also showed that R-diagonal operators and even operators, self-adjoint operators with vanishing odd moments, are related algebraically and via their free cumulants. We extend these results to the case of second order freeness. This is joint work with Octavio Arizmendi.

NORIO NAWATA, Kyushu University

Fundamental group of uniquely ergodic Cantor minimal systems

We introduce the fundamental group $\mathcal{F}(\mathcal{R}_{G,\varphi})$ of a uniquely ergodic Cantor minimal G-system $\mathcal{R}_{G,\varphi}$ where G is a countable discrete group. We compute fundamental groups of several uniquely ergodic Cantor minimal G-systems and show that if $\mathcal{R}_{G,\varphi}$ arises from a free action φ of a finitely generated abelian group, then there exists a unital countable subring R of $\mathbb{R}$ such that $\mathcal{F}(\mathcal{R}_{G,\varphi}) = R_+^\times$. Therefore $\{4^n : n \in \mathbb{Z}\}$ cannot be realized as the fundamental group of a Cantor minimal system in this class. Furthermore we consider the relation between fundamental groups of uniquely ergodic Cantor minimal $\mathbb{Z}^n$-systems and fundamental groups of crossed product C^*-algebras $C(X) \rtimes_\varphi \mathbb{Z}^n$.

MATTHIAS NEUFANG, The Fields Institute, Carleton University, Université Lille 1
Poisson boundaries over locally compact quantum groups

We present versions of several classical results on harmonic functions and Poisson boundaries in the setting of locally compact quantum groups $\mathbb{G}$. In particular, the Choquet–Deny theorem holds for compact quantum groups, and the result of Kaimanovich–Vershik and Rosenblatt, characterizing group amenability in terms of harmonic functions, admits a non-commutative analogue in the separable case. We further show that the Poisson boundary of the natural Markov operator extension of the convolution action of a quantum probability measure μ on $L_\infty(\mathbb{G})$ to $\mathcal{B}(L_2(\mathbb{G}))$, as introduced and studied – for general completely bounded multipliers on $L_1(\mathbb{G})$ – in my collaboration with M. Junge and Z.-J. Ruan, can be identified precisely with the crossed product of the Poisson boundary of μ under the coaction of $\mathbb{G}$ induced by the coproduct. This yields an affirmative answer, for general locally compact quantum groups, to a problem raised by M. Izumi (2004) in the commutative situation, in which he settled the discrete case, and unifies earlier results of W. Jaworski, V. Runde and myself. The talk is based on recent joint work with M. Kalantar and Z.-J. Ruan.

PING WONG NG, University of Louisiana at Lafayette
Commutators in the Jiang–Su algebra

Let $\mathcal{Z}$ be the Jiang–Su algebra and let τ be its unique tracial state. We prove that for all $a \in \mathcal{Z}$, the following are equivalent: (1) a is a finite sum of commutators. (2) a is a sum of five commutators. (3) $\tau(a) = 0$.

ZHUANG NIU, Memorial University of Newfoundland
Mean dimension and AH-algebras

Mean dimension for AH-algebras with diagonal maps will be discussed. It is shown that if the AH-algebra has mean dimension zero, then it must have the strict comparison on positive elements. In particular, this AH-algebra must have slow dimension growth.

CHRIS RAMSEY, University of Waterloo
Varieties and Operator Theory

To every analytic variety of the unit ball one can associate a quotient of the multiplier algebra of Drury-Arveson space. These algebras are completely isometrically isomorphic if and only if their associated varieties are biholomorphic under an automorphism of the unit ball. This is joint work with Ken Davidson and Orr Shalit.

LEONEL ROBERT, University of Louisiana at Lafayette
Remarks on stably projectionless C-algebras*

I will talk about my recent work with Bhishan Jacelon and Luis Santiago on the stably projectionless C*-algebra W, previously studied by Bhishan Jacelon. We show that W is tensorially self-absorbing. We also show that tensorial absorption of W entails strong regularity properties for a given C*-algebra. Our results lend evidence to the conjecture that amenable W-absorbing C*-algebras are classified by their cones of traces.

LUIS SANTIAGO, University of Oregon
Recovering the Elliott invariant from the Cuntz semigroup

The Cuntz semigroup has played a central role in the classification of C*-algebras. It has been used to classify simple and non-simple C*-algebras as well as to distinguish non-isomorphic C*-algebras that have the same Elliott invariant. It has been shown that for finite, nuclear, simple, unital, and $\mathcal{Z}$-absorbing C*-algebras the Elliott invariant can be recovered from the Cuntz semigroup and the K_1-group. In this talk I will explain how for this class of C*-algebras the Elliott invariant can also

be recovered from the Cuntz semigroup of the tensor product of the C*-algebra with the algebra of continuous functions on the circle.

This is a joint work with R. Antoine, M. Dadarlat and F. Perera.

DILIAN YANG, University of Windsor

Analytic free semigroup algebras are bi-operator algebras

Let $S = [S_1, ..., S_n]$ be an analytic row isometry, and $\mathfrak{S}$ the free semigroup algebra generated by S (i.e., $\mathfrak{S}$ is the unital WOT-closed algebra generated by all S_i's). We prove that $\mathfrak{S}$ is a bi-operator algebra with an integral, and that, in the category of bi-operator algebras, $\mathfrak{S}$ is completely isometrically isomorphic to $\mathcal{L}_n$, the non-commutative analytic Toeplitz algebra associated to the free semigroup on n generators. Moreover, it is shown that the predual $\mathfrak{S}_*$ of $\mathfrak{S}$ is a completely contractive abelian Banach algebra, and that there is a one-to-one correspondence between the set of all corepresentations of $\mathfrak{S}$ and the set of completely bounded representations of $\mathfrak{S}_*$.